MATH 1009 TEST 1 Solutions

<u>PART A: MULTIPLE CHOICE QUESTIONS.</u> **2** marks are given for the correct answer, **0** - otherwise. b, a, c, d, a, a, d.

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A1. The domain of the function $f(x) = \dfrac{1}{\sqrt{x-5}}$ is

(a) $[-5, 5]$, or, equivalently, $\{-5 \leq x \leq 5\}$.
(b) $(5, \infty)$, or, equivalently, $\{x > 5\}$.
(c) $[5, \infty)$, or, equivalently, $\{x \geq 5\}$.
(d) $(-\infty, 5]$, or, equivalently, $\{x \leq 5\}$.
(e) none of the above.

Answer: (b)

A2. If $f(x) = 2\sqrt{x} + 1$ and $g(x) = x^3 + 2$, then $f(g(x))$ is

(a) $2\sqrt{x^3 + 2} + 1$. **(b)** $(2\sqrt{x} + 1)^3 + 2$. **(c)** $x^3 + 2\sqrt{x} + 3$.
(d) $2x^3\sqrt{x} + 2$. **(e)** none of the above.

Answer: (a)

A3. The expression $(6^{2/3} \cdot 6^{-2})^{3/4}$ evaluates to

(a) -6. **(b)** 6. **(c)** $\dfrac{1}{6}$. **(d)** $-\dfrac{1}{6}$. **(e)** None of the above.

Answer: (c)

A4. The expression $\dfrac{e^{1.2} \cdot (e^{0.1})^{-2}}{e^{-4}}$ simplifies to

(a) $e^{4.3}$. (b) $e^{-5.7}$. (c) e^4. (d) e^5. (e) e^6.

Answer: (d)

A5. The expression $(4x^4y^6)^{1/2}$ simplifies to

(a) $2x^2y^3$. (b) $2x^2y^4$. (c) $\frac{4x^4y^6}{2}$. (d) $4x^2y^3$. (e) None of the above.

Answer: (a)

A6. The expression $\log_{\frac{1}{3}} 9$ evaluates to

(a) -2. (b) 2. (c) -1. (d) 1. (e) None of the above.

Answer: (a)

A7. Write the expression $(\ln 2 + \frac{1}{3}\ln x - 2\ln y)$ as the logarithm of a single quantity.

(a) $\ln\left(\frac{x}{3y}\right)$. (b) $\log_2\left(\frac{2x^{1/3}}{y^2}\right)$. (c) $\ln\left(2+\frac{x}{3}-2y\right)$. (d) $\ln\left(\frac{2x^{1/3}}{y^2}\right)$.

(e) None of the above.

Answer: (d)

PART B: LONG STYLE QUESTIONS.

[**7 marks**] **B1.** The weekly demand and supply equations for a company are given by $p = -2x^2 + 80$ and $p = 15x + 30$, respectively, where p is the price measured in dollars and x is measured in units **of a thousand**.

[**2**] **(a)** For the demand equation $p = -2x^2 + 80$, determine the quantity demanded, when the price is set at 8 dollars.

$$8 = -2x^2 + 80, \quad 2x^2 = 72, \quad x = \pm 6,$$

but we reject the negative root, so $x = 6$ thousand.

[**5**] **(c)** Find the equilibrium quantity and price.

At the equilibrium point, the supply is equal to the demand, and therefore

$$-2x^2 + 80 = 15x + 30.$$

Solving this equation for x yields $2x^2 + 15x - 50$, $x_{1,2} = \frac{-15 \pm 25}{4}$, $x_1 = 2.5$, $x_2 = -10$. We reject the negative root $x = -10$, since positive values of x demanded are meaningful. Thus, the equilibrium quantity is 2.5 thousand units, and the corresponding price is

$$p = -2 \cdot (2.5)^2 + 80 = 67.5$$

dollars.

[12 marks] B2. Solve each of the following equations for x.

[4] (a) $e^{2x-1} = 5$ **[4] (b)** $\log_2(x^2 - 3) = 0$ **[4] (c)** $\ln x - \ln 2 + \ln 4 = 3$

Solution:

NOTE: There is more than one way of solving each of the equations.

(a) Take the natural logarithm of both sides of the equation and use the laws of logarithms:

$$\ln(e^{2x-1}) = \ln 5 \quad \Rightarrow \quad 2x - 1 = \ln 5 \quad \Rightarrow \quad x = \frac{\ln 5 + 1}{2}.$$

(b) Since the logarithm equals zero, the argument must be equal to 1:

$$x^2 - 3 = 1 \quad \Rightarrow x^2 = 4 \quad \Rightarrow x = \pm 2.$$

(c) Simplify the LHS of the equation and then exponentiate both sides of the relation:

$$\ln\left(\frac{x \cdot 4}{2}\right) = 3 \quad \Rightarrow \quad \ln(2x) = 3 \quad \Rightarrow \quad e^{\ln(2x)} = e^3 \quad \Rightarrow \quad 2x = e^3 \quad \Rightarrow \quad x = e^3/2.$$

[7 marks] B3. The amount of \$20,000 is deposited in a bank that pays interest at the rate of 6% per year compounded **semiannually**. Using the compound interest formula

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt},$$

answer the following questions. (Round each answer to one decimal.)

[2] **(a)** What is the accumulated amount on deposit in 5 years?

$$A(5) = 20,000\left(1 + \frac{0.06}{2}\right)^{2 \cdot 5} = 20,000(1.03)^{10} = 26,878.3$$

[1] **(b)** What is the interest earned in 5 years?

$$\text{Interest } I(5) = A(5) - P = 26,878.3 - 20,000 = 6,878.3$$

[4] **(c)** How long will it take to double the investment?

$$40,000 = 20,000\left(1 + \frac{0.06}{2}\right)^{2 \cdot t} \rightarrow 2 = (1.03)^{2t} \rightarrow \ln 2 = 2t\ln(1.03) \rightarrow t = \frac{\ln 2}{2\ln 1.03} = 11.7$$