Partial Fractions

$$\int \frac{P(x)}{Q(x)}\,dx \quad , \ \deg(P) < \deg(Q)$$

① $\boxed{\text{Case 1}}$ $Q(x)$ has two real roots , $\mathcal{D} > 0$

$$\int \frac{P(x)}{Q(x)}\,dx = \underline{\quad}\ln|x-x_1| + \underline{\quad}\ln|x-x_2$$

$$\frac{P(x)}{Q(x)} = \frac{A}{x-x_1} + \frac{B}{x-x_2}$$

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② $\boxed{\text{Case 2}}$ $Q(x)$ has one double root , $\mathcal{D} = 0$

$$\int \frac{P(x)}{Q(x)}\,dx = \underline{\quad}\ln|x-x_1| - \underline{\quad}\cdot\frac{1}{x-x_1} + C$$

$$\frac{P(x)}{Q(x)} = \frac{A_1}{x-x_1} + \frac{A_2}{(x-x_1)^2}$$

③ $\boxed{\text{Case 3}}$ $Q(x)$ has no real roots, $\mathcal{D} < 0$

$$\int \frac{P(x)}{Q(x)}\,dx = \sqcup \ln|Q(x)| + \sqcup \arctan($$

(complete the square in $Q(x)$)

$\boxed{\text{Remarks}}$ $\displaystyle\int \frac{u\,du}{u^2+a^2} = \frac{1}{2}\ln|u^2+a^2|$
$+C$

$a \neq 0$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$$

Substitution technique

$$\int f[g(x)]\,g'(x)\,dx \qquad \boxed{\begin{aligned} u &= g(x) \\ du &= g'(x)\,dx \end{aligned}}$$

$$= \int f(u)\,du$$

$$\int x^{n-1} f(x^n)\,dx \quad , \; n > 1$$

$$f(x^n) = \begin{cases} \cos(x^n) \\ \sin(x^n) \\ e^{x^n} \\ \ln(x^n) \end{cases}$$

$$\int x^2 \cos(x^3)\,dx$$

$$\int \frac{1}{x} \ln^n(x)\,dx$$ [illegible] $n \neq 0$

$$\ln(x) = g(x)$$

$$\int f[g(x)] \cdot g'(x)\,dx$$

$$\int e^{10x}\left(10+e^{10x}\right)dx$$

$$\int e^{\frac{x}{10}}\left(10+e^{\frac{x}{10}}\right)dx$$

Integration by parts

$$\int x^{n-1} f(x^n)\,dx \leftarrow \text{substitution}$$

$$\int x\, f(x)\,dx \leftarrow \text{integration by parts.}$$

$$\int P(x)\, f(x)\,dx$$

$$\int \boxed{P(x)}\, e^x\,dx$$
$$\int \boxed{P(x)}\cos x\,dx$$
$$\int \boxed{P(x)}\sin x\,dx$$

$$\to u(x)$$

$$e^x dx = v' dx$$
$$e^x = v$$

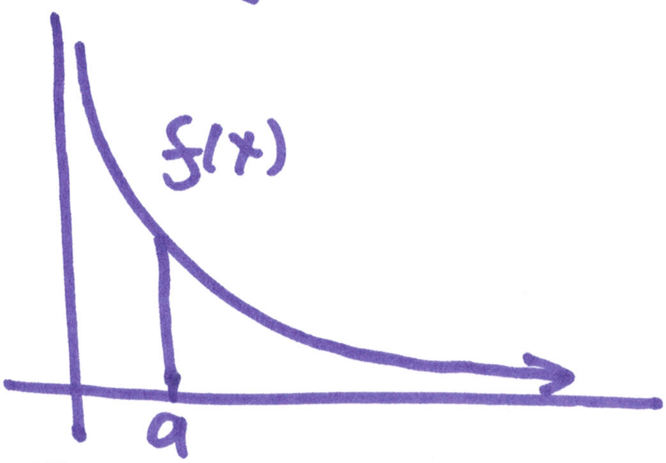
$$\int P(x)\,\boxed{\ln x}\,dx$$
$$\int P(x)\,\boxed{\arctan x}\,dx$$
$$\int P(x)\,\boxed{\arcsin x}\,dx$$

$$\downarrow$$
$$u(x)$$

$$P(x)\,dx = v'\,dx$$
$$v = \int P(x)\,dx$$

Improper Integrals

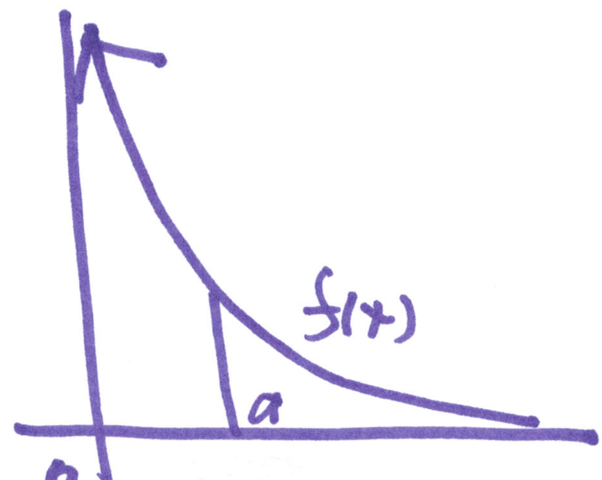
type 1



$$\int_a^\infty f(x)\,dx = \lim_{T\to\infty} \int_a^T f(x)\,dx, \quad a>0$$

$$\int_a^\infty \frac{dx}{x^p} \text{ converges if } p>1$$

type 2

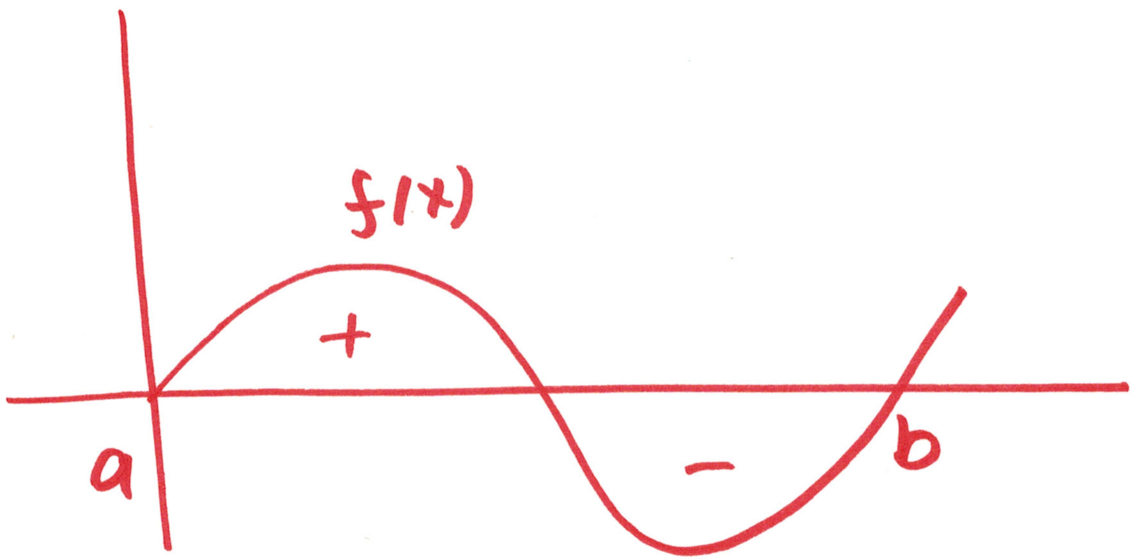


$$\int_0^a f(x)\,dx = \lim_{E\to 0^+} \int_E^a f(x)\,dx$$

$$\int_0^a \frac{dx}{x^p} \text{ converges if } p<1$$

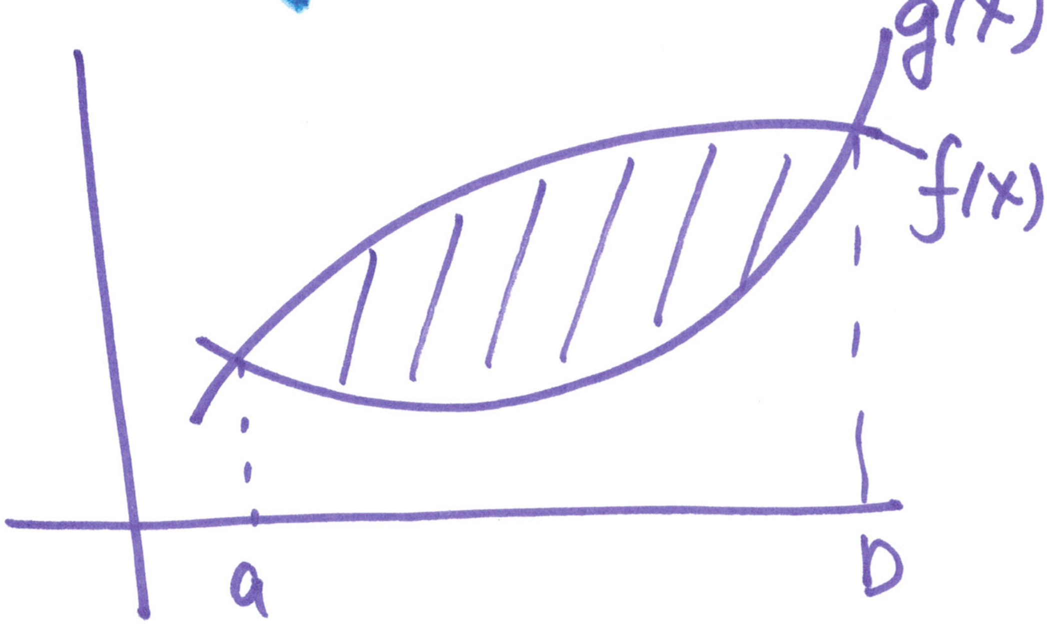
$$\int_a^b f(x)\,dx$$

an arbitrary continuous f-n

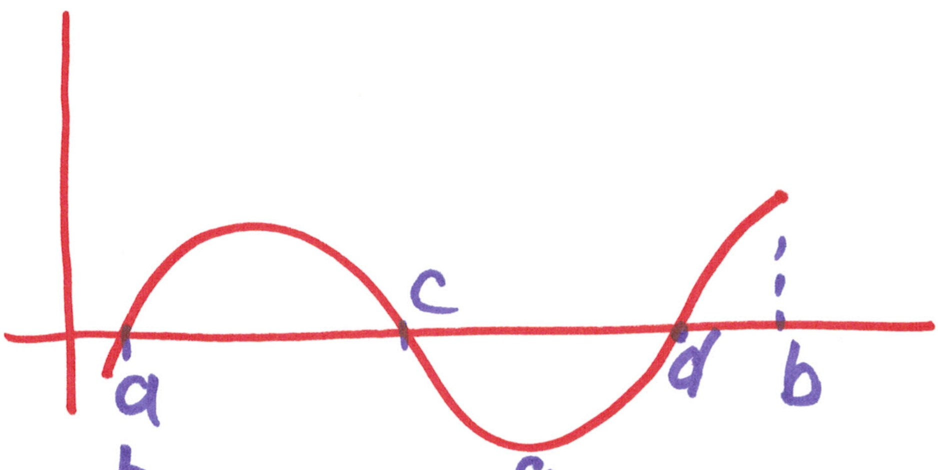


$$\int_a^b f(x)\,dx = \text{"area above the x-axis"} - \text{"area below the x-axis"}$$

Applications of definite integrals



$$\int_a^b |f(x) - g(x)|\,dx = \text{Area}$$

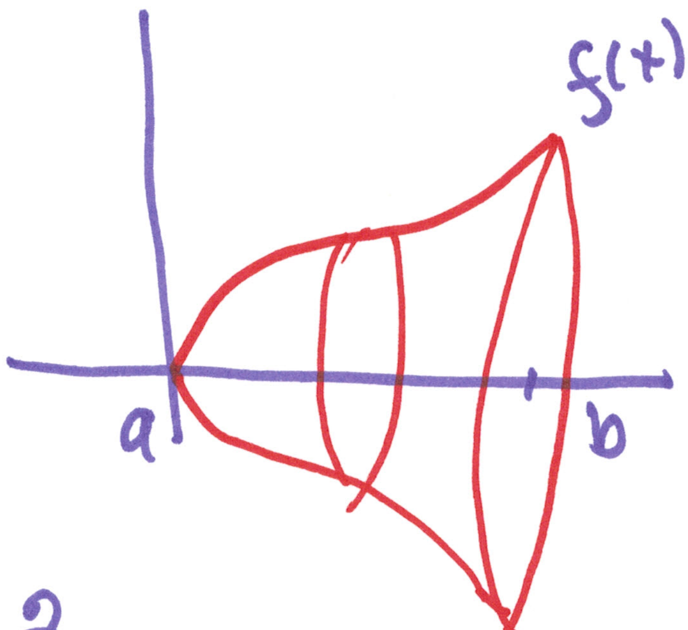


$$\text{Area} = \int_a^b |f(x)|\,dx = \int_a^c |f(x)|\,dx + \int_c^d |f(x)|\,dx +$$

$$+\int_d^b f(x)dx = \int_a^c f(x)dx + \int_c^d -f(x)dx +$$

$$+\int_d^b f(x)dx$$

Volume



$$V = \pi \int_a^b f^2(x)dx$$

Models 1 – 4

Model 1:

$$\frac{dP}{dt} = rP \nearrow P(t) = P_0 e^{rt}$$

$$P(0) = P_0$$

Model 4

$$\frac{dH}{dt} = \alpha(H - A)$$

$$H(0) = H_0$$

$$H(t) = A \pm \hat{c}\, e^{\alpha t}$$