STAT 2507 Assignment # 2 (Chapters 3&4) Winter 2013
Due in class: Sections D&E Feb. 26; Section F Feb. 27
(SOLUTION)

Last Name ——————————————- **First Name** ——————-
Student # ——————— ——————-**Lab group:**————

Total of marks=100.

Part I. Lab questions. Use only the blanks left to answer lab questions.

Download and save the accompanying data file in *labdir*.

1. [14] (Correlation Coefficient and Regression) The variable x in column $C1$ and the variable y in column $C2$, respectively, are
   x : *The number of years a faculty member has been a faculty memberat a certain University until the end of 2012*
   y : *The number of publications of each one of these faculty members utill end of 2012*

   (a) Use minitab to find the correlation Coefficient of x and y————. (In the toolbar, click on Stat→ Basic Statistics→ Correlation and type x y in the dialog box).. Sol: $r = 0.828$

   (b) According to this number what kind of relation x and y exhibit. Sol: *increasing* as $r > 0$

   (c) Use minitab (Graph→Scatterplot→With Regression) to draw the Scatter plot of x and y. Do you see a pattern? ..Sol: Yes... Describe it ..Sol: $Increasing\ and\ fairlylinear$....

   (d) Find the equation of least square regression line of y over x (response variable C2 and predictor C1). What is it? Sol: $y = 0.53 + 2.14x$

   (e) Using the regression line, find the prediction $\hat{y}$ for $x = 16$. ...Sol: $\hat{y} = 0.53 + 2.14(16) = 34.72$..

   (f) Compare the predicted value $\hat{y}$ in part (E) to actual the value of y for $x = 16$. The error is ....Sol: $34.72 - 32 = 2.72$ How close are they? ...$Fairly\ close$

(g) How many publications do you predict a person who has been a faculty member at that university for 20 years to have? Sol: $0.53 + 2.14(20) = 43.33$

2. [5] (Probability as relative frequency)
You need to use the Filliping a Fair coin applet to answer this question. This applet is available at click here.

(a) Flip the coin 10 times what is $\frac{\#\ of\ Heads\ observed}{total\ \#\ of\ flips}$?———— Sol: I had 4/10=0.4———-

(b) (press Reset) Flip the coin 100 times. What is $\frac{\#\ of\ Heads\ observed}{total\ \#\ of\ flips}$? ———— Sol: I had 53/100=0.53————

(c) (Press Reset) By pressing Flip for $n = 1000$ once, flip the coin 1000 times. What is $\frac{\#\ of\ Heads\ observed}{total\ \#\ of\ flips}$? ———— Sol: I had 503/1000 ————

(d) Compare the relative frequencies in part (a), part (b) and part (c). Which one is closer to 0.5?————- Sol: part (c) ————

(e) Suppose that you could flip it infinitely many times, then what the value of $\frac{\#\ of\ Heads\ observed}{total\ \#\ of\ flips}$ would be? ————Sol: 1/2 ————

3. [5] (Conditional probability and Independence) the link click here gives an experiment of tossing two fair dice (one green and one red).
Let C be the event that the green die shows a number less than or equal to 2.
Let D be the event that the red die shows a number less than or equal to 3.

(a) (Press Reset) Obtain $P(C)$= ————Sol: 12/36——— –

(b) (Press Reset) Obtain $P(D)$= ————-Sol: 18/36 ————

(c) (Press Reset) By clicking on the simple events in which the green die has a number at most 2 and the red one has a number at most 3, obtain $P(C \cap D)$= ————-Sol: 6/36 ————

(d) (Use parts (b) and (c)) If you know that the red die showed a number less than or equal to 3, then obtain the probability that the green shows a number less than or equal to 2, by clicking on the relevant simple events in the applet?
I.e., what is $P(C|D)$ = ————Sol: 6/18 ———-

(e) What is your conclusion about the relation between the events C and D? ————Sol: Independent as $P(C|D) = P(C)$ ———

4. [10] (Mean and variance of random variables using Minitab)

   (a) Obtain the mean of the random variable whose values are in $C6$, i.e., $\mu = E(x)$ by $Calc \rightarrow Calculator \rightarrow sum(C6 * C7)$. What is μ?——Sol: 6.5

   (b) by clicking $Calc \rightarrow Calculator \rightarrow sum((C6 * *2) * C7) - (sum(C6 * C7)) * *2$. obtain $Var(x) = \sigma^2$. What is σ^2 ?——Sol: 3.25 What is σ?- sol: 1.802

   (c) Calculate the interval $(\mu - 2\sigma, \mu + 2\sigma)$? ——Sol: $(2.9, 10.1)$

   (d) What portion of the values of the random variable lie inside the interval in part (c)? —— Sol: $8/14 \approx .57$

Part II Comprehension questions

1. A student is preparing for an upcoming exam. The professor for the course has given the class 30 questions to study from and plans to select 10 of the questions for use on the actual exam. Suppose that the student knows how to solve 25 of the 30 questions.

   (a) [4] What is the probability that the student will get perfect on the test?

   (b) [6] What is the probability that the student will get at least 9 questions correct on the test?

   Solution: (a) $\frac{\binom{25}{10}}{\binom{30}{10}}$.

   Solution: (b) $\frac{\binom{25}{9}\binom{5}{1}}{\binom{30}{10}} + \frac{\binom{25}{10}}{\binom{30}{10}}$

2. [10] Twenty persons reporting to a Red Cross center one day are typed for blood, and the following counts are found:

Blood group	O	A	B	AB	Total
No. of persons	7	4	6	3	20

   If one person is randomly selected, what is the probability that this person's blood group is:

   (a) AB?

   (b) Either A or B?

(c) O and B?

(d) Not O?

(e) A given that his blood is Not O?

SOLUTION
(a) $Pr(AB) = \frac{3}{20}$
(b) $Pr(A \cup B) = \frac{4+6}{20}$
(c)$Pr(O \cap B) = \frac{0}{20} = 0$
(d) $Pr(Not\ O) = 1 - Pr(O) = 1 - \frac{7}{20} = \frac{13}{20}$
(e)$Pr(A|\{NotO\}) = \frac{Pr(A \cap \{NotO\})}{Pr(\{NotO\})} = \frac{\frac{4}{20}}{\frac{13}{20}} = \frac{4}{13}$

3. [10] If 5 balls are thrown at random into 10 boxes, what is the probability that no box will receive more than one ball?
SOLUTION There are 10^5 possible outcomes in the sample space, i.e., 10 boxes for each one of the 5 balls. If the 5 balls are to be thrown into different boxes (this is equivalent to choosing 5 boxes, for the five balls, out of 10 boxes), there are C_5^{10} ways to do that. So, the probability is $\frac{C_5^{10}}{10^5}$.

4. A survey has revealed that 75% of all college students study. It is also known that 85% of all students who study will graduate, while only 35% of those students who do not study will graduate.

   (a) [6] If a student is randomly selected, what is the probability that he or she will graduate?

   (b) [8] A randomly selected student is observed to graduate. What is the probability that this student studied?

Solution: (a) Let $G = \{student\ graduates\}$;
$H = \{student\ doesn?t\ graduate\}$;
$S = \{student\ studies\}$;
$T = \{student\ doesn?t\ study\}$
$P(G) = P(G|S)P(S) + P(G|T)P(T) = 0.85 \times 0.75 + 0.35 \times (1 - 0.75) = 0.725$
(b) Using Bayes' rule: $P(S|G) = \dfrac{P(G|S)P(S)}{P(G|S)P(S) + P(G|T)P(T)} = \dfrac{0.85 \times 0.75}{0.725} = 0.879.$

5. For events A and B we have

$$P(A) = 0.3,\ P(B) = 0.8,\ P(A \cup B) = 0.9$$

(a) [9] Find $P(A|B)$, $P(A' \cap B)$ and $P(B' \cup A')$.

(b) [3] Are A and B independent? Why?

Solution: (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) \rightarrow P(A \cap B) = 0.3 + 0.8 - 0.9 = 0.2$
$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.8} = 0.25$
$P(A' \cap B) = P(B) - P(A \cap B) = 0.8 - 0.2 = 0.6$
$P(B' \cup A') = 1 - P(A \cap B) = 1 - 0.2 = 0.8$
(b) No there are not independent, since
$P(A \cap B) \neq P(A)P(B), i.e.,\ 0.2 \neq (0.3)(0.8)$

6. A random variable X can assume five values: $0, 1, 2, 3, 4$. A portion of the probability distribution is shown here:

x	0	1	2	3	4
p(x)	0.2	0.1	0.1	?	0.3

(a) [2] Find p(3).

(b) [4] Calculate the population mean, variance, and standard deviation.

(c) [2] What is the probability that X is strictly greater than 1;

(d) [2] What is the probability that X is less than 3 and greater than 1;

(In parts (a)-(d) show steps of your calculations)

SOLUTION

$$(a)\ p(3) = 1 - (0.2 + 0.1 + 0.1 + 0.3) = 0.3.$$

$$(b)\ \mu = 0 \times 0.2 + 1 \times 0.1 + 2 \times 0.1 + 3 \times 0.3 + 4 \times 0.3 = 2.4$$

$$\sigma^2 = (0 - 2.4)^2 \times 0.2 + (1 - 2.4)^2 \times 0.1 + (2 - 2.4)^2 \times 0.1 + (3 - 2.4)^2 \times 0.3 + (4 - 2.4)^2 \times 0.3 = 2.24$$

$$\sqrt{\sigma^2} = 1.4967$$

$$(c)\ Pr(X > 1) = 0.1 + 0.3 + 0.3 = 0.7$$

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$$(d)\ Pr(1 < X < 3) = 0.1$$