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(**Do not** write your ID number on this page; your ID number goes on all pages after this one.)

PHYSICS 259 Midterm Test Solutions

February 17, 2011 90 minutes

DO NOT TEAR OFF THIS PAGE! You may, however, tear off the last page.

This test consists of twelve (12) pages. Please ensure that you have all twelve pages.

Do all questions. Use of the Schulich-approved calculator is permitted. There is an equation sheet and table of integrals on the back of the final page.

All answers for marking must be written on the ANSWER PAGES provided. Rough work may be written on this question paper, and will not be marked.

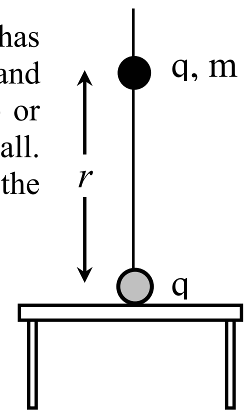
Part A of the exam consists of 10 multiple-choice questions, worth one mark each. **We recommend that you take no more than 30 minutes for this part of the exam.** Many of the questions require little or no calculations. Read each question carefully. If you get stuck ... move on. There is only one answer which will be accepted as correct for each question.

Part B of the exam consists of four "long-answer" questions worth a total of 21 marks. **We recommend that you take at least one hour for this part of the exam.** For full marks, ***all work must be shown*** **and** ***all reasoning clearly explained.*** Show all equations in symbols first, defining symbols. Where numbers are used, show how numbers are substituted into equations using correct units.

Assume all numbers are accurate to three significant figures unless more figures are specified.

PART A. Multiple-choice questions. For each question, **mark your choice for the correct answer on the first page of the ANSWER SHEETS.** One mark each.

1. In the figure at the right, a small, grey ball is fixed to a tabletop and has been given a charge q. A vertical thread runs upward from this ball, and a black ball of mass m = 2.04 g is free to slide without friction up or down this thread. The black ball has the same charge, q, as the grey ball. If the equilibrium distance, *r*, between the balls is 0.500 m, what is the value of q? Choose the closest answer.



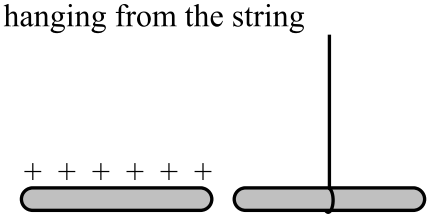
   a. 1.80 μC.
   b. 4.24 μC.
   c. 1.06 μC.
   d. 0.746 μC.
   e. 2.12 μC.

Reasoning: *The black ball is in equilibrium when the upward Coulomb force on it exactly balances the downward gravitational force on it (i.e., its weight). Then*

$$\frac{1}{4\pi\varepsilon_0}\frac{q\times q}{r^2} = mg$$

$$q = \sqrt{4\pi\varepsilon_0 mgr^2} = \sqrt{4\pi(8.85\times10^{-12}\,C^2/Nm^2)(0.00204\,kg)(9.81\,m/s^2)(0.500\,m)^2}$$
$$= 7.46\times10^{-7}\,C = \underline{\underline{0.746\,\mu C}}$$

2. A positively-charged, insulating rod is brought close to (but not touching) one end of a second rod that is hanging from a string. The charged rod attracts the rod on the string. From this observation we can be absolutely certain that the rod hanging from the string

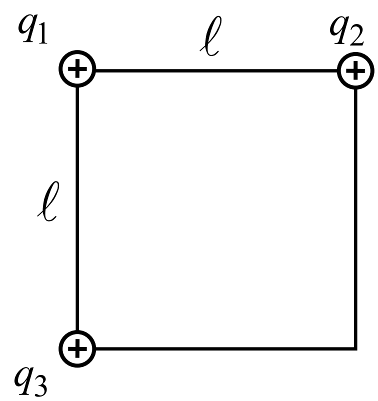


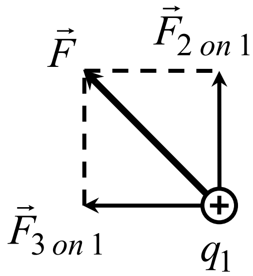
   a. is a conducting rod.
   b. is an insulating rod.
   c. has a net charge.
   d. does not have a net charge.
   e. We are not certain of any of a, b, c or d.

Reasoning: *Consider first the case where the hanging rod is uncharged. The charged rod attracts the negative charges in the hanging rod and repels the positive charges. Thus, the negative charges in the hanging rod are closer to the charged rod and the positive charges are further away, resulting in a net attraction. If the hanging rod is conducting, then there is greater separation of charge and the attraction is strong, and if the hanging rod is insulating then the charge separation occurs only within individual molecules and the attraction is weaker, but in both cases attraction occurs. Therefore, we cannot decide between answers a and b.*

The hanging rod could be charged negatively or it could be uncharged, and it will still be attracted, so the attraction does not allow us to decide between answers c and d.

3. Charges $q_1 = q_2 = q_3 = 8.00\ \mu C$ are at the corners of a square of sides $\ell = 0.200$ m, as shown at the right. What is the magnitude of the force on q_1 by the other two charges?





a. 14.4 N.
b. 20.3 N.
c. 28.8 N.
d. 4.07 N.
e. 5.75 N.

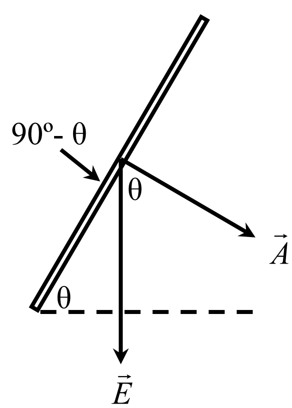
Reasoning: *The magnitudes of* $\vec{F}_{2\ on\ 1}$ *and* $\vec{F}_{3\ on\ 1}$ *are equal:*

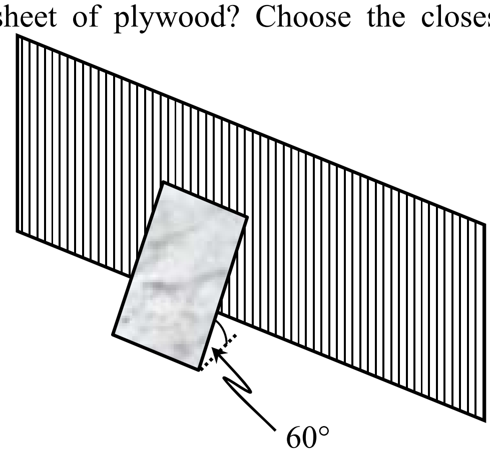
$$F_{2\ on\ 1} = F_{3\ on\ 1} = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{r^2} = \frac{1}{4\pi(8.85\times10^{-12}\ C^2/Nm^2)}\frac{(8.00\times10^{-6}\ C)^2}{(0.200\ m)^2} = 14.4\ N$$

Then the magnitude of $\vec{F}$ *is* $F = \sqrt{(F_{2\ on\ 1})^2 + (F_{3\ on\ 1})^2} = \sqrt{2\,(14.4\ N)^2} = \underline{\underline{20.3\ N}}$

4. A sheet of plywood of area 3.00 m^2 is leaning against the wall of a shed. The angle between the plywood and the ground is 60º. If there is an electric field of 150 N/C directed downward, what is the electric flux through the sheet of plywood? Choose the closest answer.

a. 450 $N\,m^2\,C^{-1}$
b. 390 $N\,m^2\,C^{-1}$
c. 225 $N\,m^2\,C^{-1}$
d. 0.
e. 900 $N\,m^2\,C^{-1}$





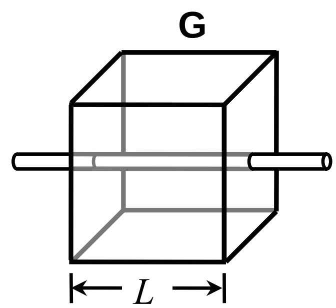
Reasoning:

The angle between the vectors $\vec{E}$ *and* $\vec{A}$ *is equal to the angle between the plywood and the ground (see figure above). Then*

$$\Phi_E = \vec{E}\bullet\vec{A} = EA\cos\theta = (150\ N/C)(3.00\ m^2)\cos 60° = \underline{\underline{225\ Nm^2/C}}$$

5. In the diagram below, a horizontal, insulating rod of length 2L carries a uniform, positive linear charge density (charge per unit length) λ. A student has drawn a Gaussian surface, labelled **G**, in the shape of a cube with edges of length L, centred on the rod.

The net outward flux through this closed Gaussian surface is

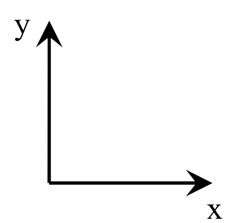


a. $2L\lambda/\varepsilon_0$.
b. impossible to calculate.
c. $L\lambda/\varepsilon_0$.
d. $6L^2\lambda/\varepsilon_0$.
e. $4L^2\lambda/\varepsilon_0$.

Reasoning: *From Gauss's law, the net outward flux through the cube equals the charge enclosed by the cube, divided by ε_0. A length L of the rod is enclosed within the Gaussian surface and the charge per unit length is λ, so the net outward flux is L λ/ε_0.*

6. The electric potential in a particular area is given by $V(x,y) = -2xy + y^2$, where V is in volts, x and y are in metres, and the x and y axes are directed as indicated below. What is the direction of the electric field at the point x = 1.0 m, y = 3.0 m?

a. Toward the upper right.
b. Toward the lower right.
c. Toward the upper left.
d. Toward the lower left.
e. Vertically downward.

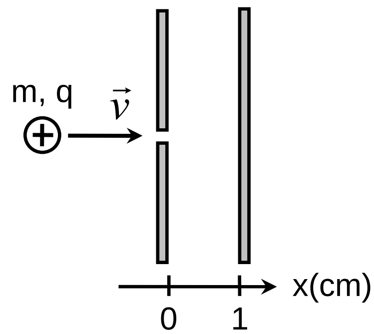


Reasoning: *The x- and y-components of $\vec{E}$ are*

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(-2xy + y^2) = +2y = 2(3) = 6\,N/C$$
$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(-2xy + y^2) = 2x - 2y = 2(1) - 2(3) = -4\,N/C$$

E_x is positive and E_y is negative, so $\vec{E}$ points toward the lower right.

7. The figure below shows a side view of two metal plates 1.00 cm apart. The plates are large enough that the electric field between them is uniform. The potential difference between the plates is 80.0 V, with the right-hand plate being at the more positive potential. A proton (mass m = 1.67 x 10^{-27} kg, charge q = +e) travels through a small hole in the left-hand plate at a speed v = 1.00 x 10^5 m/s. What happens to the proton? Select the best answer.



a. It turns back at x = 0.65 cm.
b. It *just* reaches the other plate (v = 0 at x = 1.0 cm).
c. It turns back immediately (x = 0) because it cannot enter a region of opposing electric field.
d. It turns back at x = 0.35 cm.
e. It reaches the other plate at a speed v = 1.6 x 10^5 m/s.

Reasoning: *Either it reaches the other plate or it doesn't. If it does, then we can use conservation of mechanical energy to find its speed at the other plate. If it doesn't, then we can use conservation of mechanical energy to find how far it travels before turning back. First, try assuming that it reaches the other plate:*

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}mv_f^2 + qV_f = \tfrac{1}{2}mv_i^2 + qV_i$$

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 + qV_i - qV_f = \tfrac{1}{2}mv_i^2 + q(V_i - V_f) = \tfrac{1}{2}mv_i^2 + q(V_{i,f}) \qquad \textbf{(1)}$$

where $V_{i,f}$ *is the potential difference between the initial and final points. Then*

$$v_f = \sqrt{v_i^2 + \frac{2q}{m}(V_{i,f})} = \sqrt{(10^5\ m/s)^2 + \frac{2(1.60\times10^{-19}\ C)}{1.67\times10^{-27}\ kg}(-80V)}$$
$$= \sqrt{10^{-10}\ m^2/s^2 - 1.53\times10^{-10}\ m^2/s^2}$$

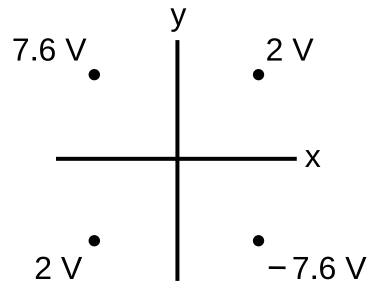
The quantity under the square root sign is negative, which means that the proton does not make it to the opposite plate. Then from ***(1)****, the potential difference through which it does travel is*

$$V_{i,f} = \frac{\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2}{q} = \frac{m(v_f^2 - v_i^2)}{2q} = \frac{(1.67\times10^{-27}\ kg)(0-(10^5\ m/s)^2)}{2(1.60\times10^{-19}\ C)} = -52.2V$$

The electric field is uniform, so potential changes linearly and the distance the proton travels between the plates is

$$x = \frac{52.2V}{80V}(1\,cm) = \underline{\underline{0.65\,cm}}$$

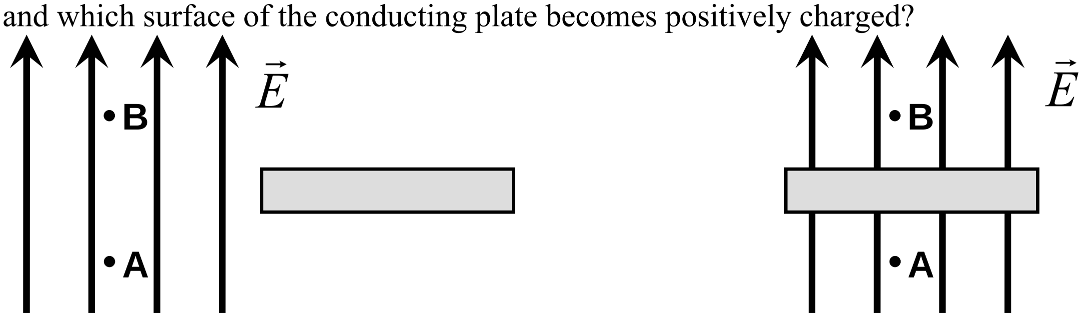
8. A person has measured the electric potential at four locations in the (x, y) plane (see figure below). Assuming that the z-component of the electric field is zero, what is the direction of the electric field relative to the axes shown?



a. Toward the upper right.
b. Toward the lower left.
c. Toward the upper left.
d. Toward the lower right.
e. The electric field is zero.

Reasoning: *The two points at 2V potential define an equipotential line from lower left to upper right, and the electric field vector (or field line) is perpendicular to the equipotential. This tells us that the electric field direction is either toward the upper left or toward the lower right. The next point to remember is that the electric field is always directed from higher potential toward lower potential. The 7.6 V and -7.6 V potentials then tell us that the electric field direction is from the upper left toward the lower right.*

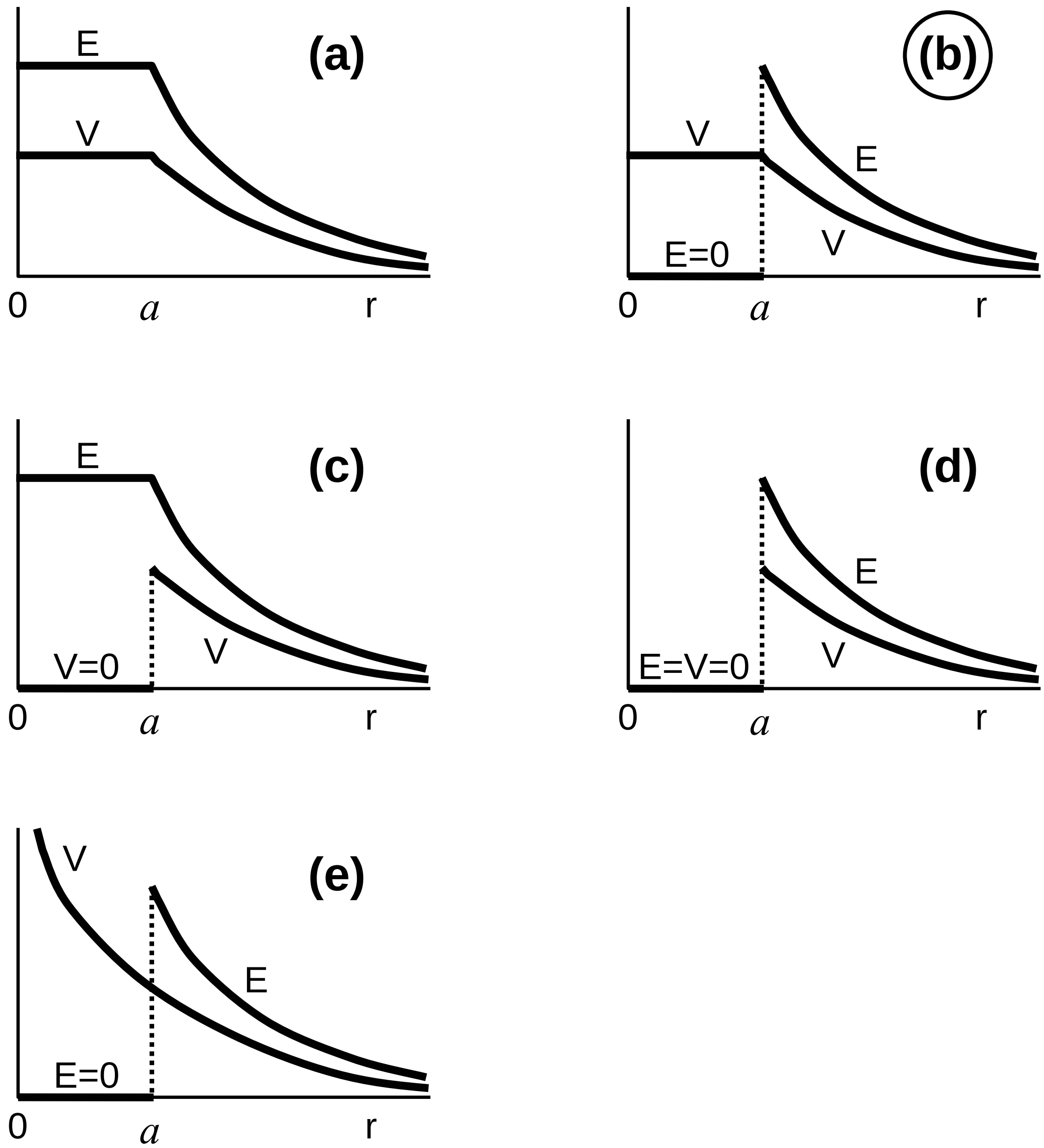
9. In the left-hand figure, below, points A and B are located in a region of uniform electric field, $\vec{E}$. An uncharged, thick, conducting plate (grey rectangle) is now slid between points A and B, as shown in the right-hand figure, while the electric field strength, E, is maintained at a constant value. What happens to the potential difference, V_{AB}, between points A and B, and which surface of the conducting plate becomes positively charged?



a. V_{AB} decreases and the top surface of the plate becomes positively charged.
b. V_{AB} decreases and the bottom surface of the plate becomes positively charged.
c. V_{AB} remains the same and the top surface of the plate becomes positively charged.
d. V_{AB} remains the same and the bottom surface of the plate becomes positively charged.
e. V_{AB} increases and the bottom surface of the plate becomes positively charged.

Reasoning: *The potential difference V_{AB} is given by $V_{AB} = \int_A^B \vec{E} \bullet d\vec{\ell}$. Integration is just adding things up, so what we do is to walk from A to B, adding up $\vec{E} \bullet d\vec{\ell}$ as we go. In the left-hand diagram, E is non-zero all the way from A to B, whereas in the right-hand diagram, there is a part of the path (inside the conductor) where E equals zero, but outside this, E is the same as in the left-hand diagram. Therefore, the sum of all the $\vec{E} \bullet d\vec{\ell}$ is smaller in the RH diagram than in the LH diagram; i.e., V_{AB} decreases when we insert the conductor. The force on the positive charges inside the conductor is in the direction of the electric field, so the top surface of the plate becomes positively charged.*

10. A solid metal sphere of radius *a* has been given an excess positive charge. Which of the five graphs, below, shows the behaviour of the electric potential, V (relative to V = 0 at infinity), and the electric field strength, E, inside and outside this sphere as a function of the distance, r, from the centre of the sphere?



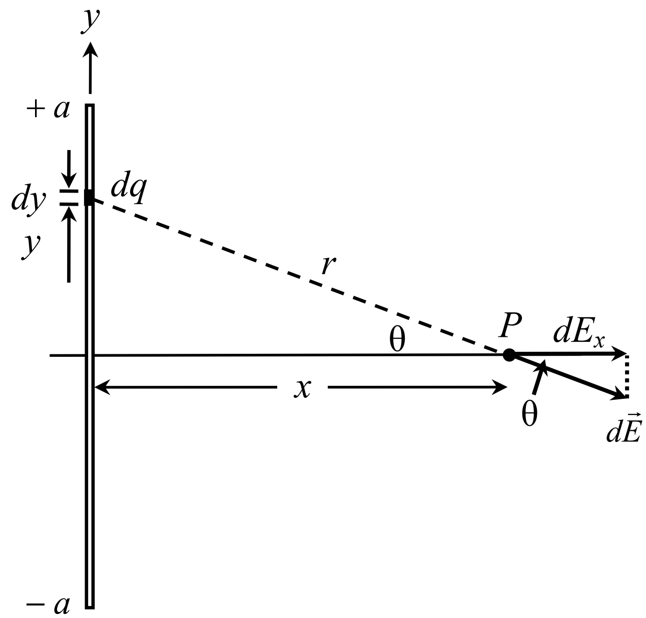
Reasoning: *The electric field inside a conductor is zero in a static situation, so E = 0 at r<a. The potential* ***difference*** *between the centre (at r=0) and the surface at r=a) is given by*

$$V_{0,a} = \int_{r=0}^{a} \vec{E} \bullet d\vec{\ell}$$

If E=0 then the integral is zero, so there is no potential ***difference*** *between the centre and the surface of the sphere; i.e., the entire sphere is a 3-dimensional equipotential volume, and V at the centre equals V at the surface.*

PART B. Long-answer questions. Do all questions. Rough work may be done on this question paper, and will not be marked. ***Finished solutions must be shown on the ANSWER PAGES. For full marks, show all work clearly and explain all reasoning.***

12. In the figure at the right, a thin, insulating rod of length 2*a* lies along the y-axis with its centre at y = 0. Positive charge is distributed uniformly along the rod with a linear charge density (charge per unit length) λ.



1 mark a. How much charge, dq, is contained within any infinitesimally-short length, dy, of the rod? (See the figure at the right.) Express your answer in terms of dy and any other necessary quantities.

Solution: *The amount of charge contained in any length dy equals the amount of charge per unit length times the length:*

$$\boxed{dq = \lambda dy.}$$

2 marks b. Find the x-component of the electric field vector that the small charge, dq, creates at point *P* in the figure above. Express your answer in terms of *x*, *y*, λ, and constants.

Solution: *In the diagram above, the infinitesimal charge dq behaves as a point charge, so it creates an infinitesimal electric field* $d\vec{E}$ *at point P of magnitude*

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

The x-component of $d\vec{E}$ *is* $dE_x = dE\cos\theta$ *where* $\cos\theta = \frac{x}{r}$. *Then*

$$dE_x = dE\cos\theta = \left(\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\right)\left(\frac{x}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{xdq}{r^3}.$$

Also, $dq = \lambda dy$ *from Part a, and* $r = \sqrt{x^2 + y^2} = (x^2 + y^2)^{\frac{1}{2}}$.

Then
$$\boxed{dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x dy}{(x^2 + y^2)^{\frac{3}{2}}}.}$$

1 mark c. What is the total y-component, E_y, of the electric field vector at point *P* due to all of the charge on the rod shown above? Explain your answer in two sentences or less.

Solution: *E_y = 0, by symmetry. If we consider an equal charge, dq, placed symmetrically below the axis in the previous figure, then the y-components of the two fields add to zero. The sum of the fields due to all such pairs is therefore also zero.*

2 marks d. Find the magnitude of the total electric field, E, at point *P* due to all of the charge on the rod shown above. Express your answer in terms of a, x, λ, and constants.

Solution: *The total field is obtained by adding up all the little bits of field; i.e., integrating. These bits of field arise from source charges distributed from y = -a to y = a. Using the result from Part b:*

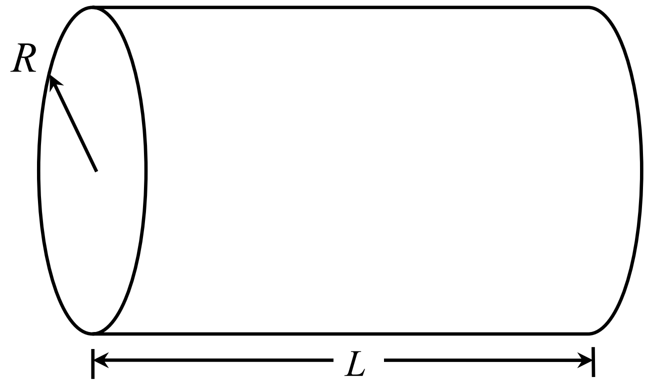
$$E = \int_{y=-a}^{a} dE_x = \int_{y=-a}^{a} \frac{1}{4\pi\varepsilon_0} \frac{\lambda x dy}{(x^2 + y^2)^{\frac{3}{2}}} = \frac{\lambda x}{4\pi\varepsilon_0} \int_{y=-a}^{a} \frac{dy}{(x^2 + y^2)^{\frac{3}{2}}} = \frac{\lambda x}{4\pi\varepsilon_0} \frac{1}{x^2} \frac{y}{\sqrt{x^2 + y^2}}\Bigg|_{y=-a}^{a}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 x}\left[\frac{a}{\sqrt{x^2 + a^2}} - \frac{-a}{\sqrt{x^2 + a^2}}\right] = \frac{\lambda}{4\pi\varepsilon_0 x} \frac{2a}{\sqrt{x^2 + a^2}}$$

So

$$\boxed{E = \frac{\lambda a}{2\pi\varepsilon_0 x\sqrt{x^2 + a^2}}}$$

13. A very long, solid, insulating cylinder of radius R contains a uniform volume charge density ρ. The figure at the right shows a segment of length L, but the actual cylinder extends to large distances toward the left and right.



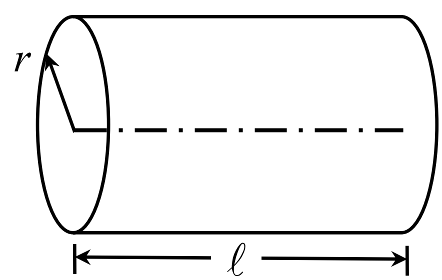
1 mark a. What total charge, Q, is contained in this segment of the cylinder? Express your answer in terms of quantities given in the problem, and constants.

Solution: *The total charge equals the charge per unit volume times the volume, and the volume of a cylinder is the cross-sectional area times the length. The cross-section is a circle, so*

$$\boxed{Q = \rho V = \rho A L = \rho \pi R^2 L.}$$

2 marks b. ***Write down Gauss's law*** and use it to derive the magnitude of the electric field, E, as a function of the radial coordinate, r, inside the long cylinder; i.e., in the region $0 \le r \le R$. Show your steps and reasoning, and make a drawing of the Gaussian surface you are using. Express your answer in terms of quantities given in the problem, and constants.

Solution: *Gauss's law is* $\Phi_E = \oint \vec{E} \bullet d\vec{A} = \dfrac{Q_{encl}}{\varepsilon_0}$



For a Gaussian surface, use a smaller cylinder of arbitrary radius, r, and length ℓ, inside and coaxial with the long cylinder. On the two ends, $\vec{E} \perp d\vec{A}$ so the dot product is zero, and on the sides $\vec{E} \parallel d\vec{A}$. Also, as we walk around on the sides, we are always the same distance from the central axis, so by cylindrical symmetry we expect that the magnitude, E, is constant. Then

$$\oint \vec{E} \bullet d\vec{A} = \int_{sides} E\, dA = E \int_{sides} dA = EA_{sides} = E\, 2\pi r \ell.$$

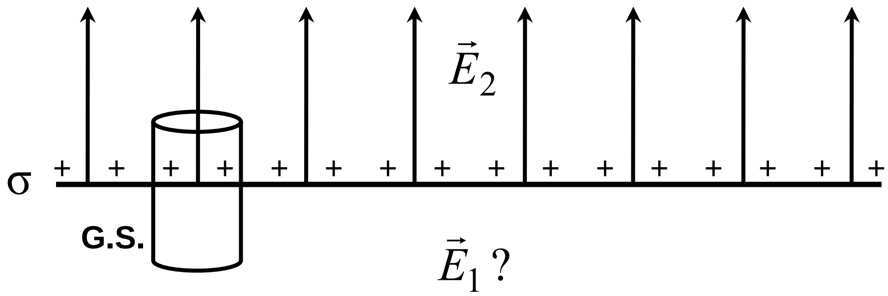
From Part a, the charge enclosed by the cylinder is $Q_{encl} = \rho \pi r^2 \ell$. Then

$$E\, 2\pi r \ell = \rho \pi r^2 \ell$$

and

$$\boxed{E = \frac{\rho r}{2}}$$

2 marks c. *(Part c is not related to parts a or b.)* The horizontal line in the figure below represents a very large, very thin, horizontal sheet that carries a total charge per unit area $\sigma = 2.66 \times 10^{-9}$ C/m^2. (There may be other charges outside the figure that are not shown.) The electric field in the space above the sheet is $E_2 = 1200$ N/C. What are the magnitude and direction of the electric field, $\vec{E}_1$, in the space below the sheet?



Solution: *We know that the electric field of the sheet itself is perpendicular to the sheet. The field lines above the sheet are perpendicular to the sheet, so any external field (due to charges outside the figure) has no horizontal component. Therefore, the electric field below the sheet must be perpendicular to the sheet as well (or else zero).*

Choose a cylindrical Gaussian surface (G.S.) perpendicular to the sheet as shown above. Then only the top and bottom ends contribute to the flux, because $\vec{E} \perp d\vec{A}$ on the sides. We don't know the direction of $\vec{E}_1$, so assume it is downward to make the flux positive on the bottom end of the cylinder. (If we get a negative value for E_1, then we simply chose the incorrect direction and E_1 must be upward.) Also, if the field is purely upward then both fields are uniform. If the cross-sectional area of the GS is A, then the LHS of Gauss's law gives

$$\oint \vec{E} \bullet d\vec{A} = \int_{bottom\ end} E_1\, dA + \int_{top\ end} E_2\, dA = E_1 \int_{bottom\ end} dA + E_2 \int_{top\ end} dA = E_1 A + E_2 A$$

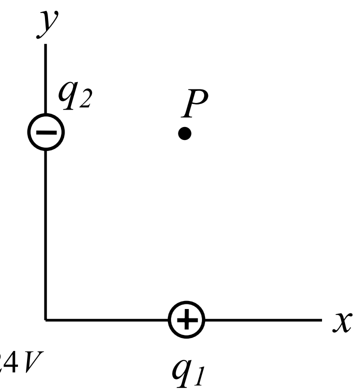
The charge enclosed by the G.S. is $Q_{encl} = \sigma A$.

Then $E_1 A + E_2 A = \dfrac{\sigma A}{\varepsilon_0}$

$$E_1 = \frac{\sigma}{\varepsilon_0} - E_2 = \frac{2.66 \times 10^{-9}\, C/m^2}{8.85 \times 10^{-12}\, C^2/Nm^2} - 1200\, N/C = 300\, N/C - 1200\, N/C = -900\, N/C.$$

The negative value tells us we chose the incorrect direction, so E_1 = __900 N/C upward__.

14. In the figure at the right, a point charge $q_1 = +5.00$ nC is located on the x-axis at $x = 3.00$ cm, and a second point charge $q_2 = -3.00$ nC is located on the y-axis at $y = 4.00$ cm.



2 marks a. Find the electric potential, V, at point P at the position (x,y) = (3.00 cm, 4.00 cm). Assume V = 0 at infinity.

Solution:

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi(8.85\times10^{-12}\,C^2/Nm^2)}\frac{5.00\times10^{-9}\,C}{0.04\,m} = 1124\,V$$

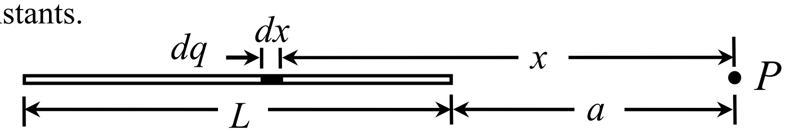
$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} = \frac{1}{4\pi(8.85\times10^{-12}\,C^2/Nm^2)}\frac{-3.00\times10^{-9}\,C}{0.03\,m} = -899\,V$$

Then $V = V_1 + V_2 = 1124\,V - 899\,V = \underline{\underline{225\,V}}.$

1 mark b. A third charge, $q_3 = +2.00$ nC, is now placed at point P. What is the potential energy of q_3 relative to the other two charges?

Solution: $U_3 = q_3 V = (2.00\times10^{-9}\,C)(225\,V) = \underline{\underline{4.50\times10^{-7}\,J}}.$

2 marks c. *(Part c is not related to parts a or b.)* The figure below shows a thin, insulating rod of length L. Positive charge is distributed uniformly along the rod with a linear charge density (charge per unit length) λ. Find the electric potential, V, at point P on the axis of the rod at a distance a from the end of the rod, as shown. Express your answer in terms of L, a, λ and constants.



Solution: *Choose an infinitesimal charge, dq, of length dx at a distance x from point P, as shown above. This charge creates an infinitesimal potential at P equal to*

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x}$$

The amount of charge in length dx is λdx, so $dV = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda dx}{x}.$

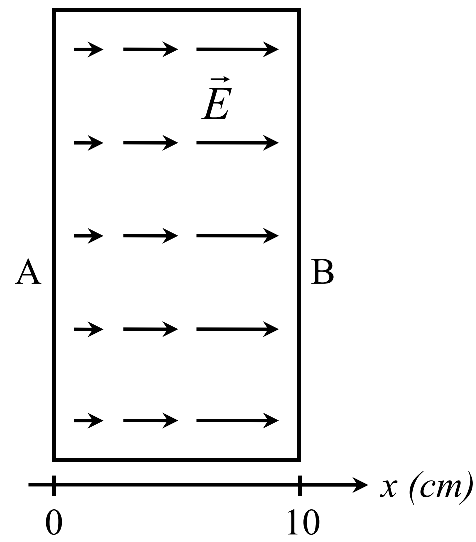
Then to find the total potential at point P, just add up all the little bits of potential due to all of the dq along the rod:

$$V = \int_{x=a}^{a+L} dV = \int_{x=a}^{a+L} \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x} = \frac{\lambda}{4\pi\varepsilon_0}\int_{x=a}^{a+L}\frac{dx}{x} = \frac{\lambda}{4\pi\varepsilon_0}\ln x\Big|_{x=a}^{a+L} = \boxed{\frac{\lambda}{4\pi\varepsilon_0}\ln\frac{a+L}{a}}.$$

15. The figure at the right shows a thick slab of insulating material of thickness L = 10.0 cm, with its left side at x = 0. The electric field inside the slab is given by

$$\vec{E} = Cx^2\,\hat{i}$$

where C is a constant. The potential difference between sides A and B is 75.0 V.



2 marks a. Find the numerical value of the constant, C, with correct units. [HINT: How are electric field and potential difference related?]

Solution: *We know the potential difference and we have an equation for the electric field, so we need to relate the two:*

$$V_{ab} = \int_a^b \vec{E} \bullet d\vec{l}$$

Choose a path that follows an electric field line; then $d\vec{\ell} \parallel \vec{E}$ *and* $d\ell = dx$. *Then*

$$75V = \int_a^b E dx = \int_{x=0}^{0.1\,m} Cx^2 dx = C\int_{x=0}^{0.1\,m} x^2 dx = C\frac{x^3}{3}\Bigg|_{x=0}^{0.1\,m} = \frac{10^{-3}\,m^3}{3}C$$

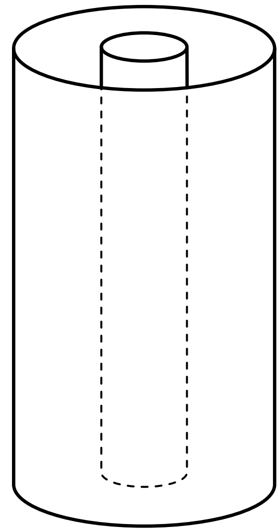
and

$$C = \frac{3(75V)}{10^{-3}\,m^3} = \underline{\underline{2.25\times 10^5\,\frac{V}{m^3}}}$$

1 mark b. Which side of the block (side A or side B) is at the higher potential?

Solution: *Electric field always points from higher to lower potential, so* <u>*side A*</u> *is at the higher potential.*

2 marks c. *(Parts c and d are not related to parts a and b.)* The two metal cylinders in the figure at the right are co-axial and are separated by vacuum. The inner cylinder has radius r_a = 3.00 mm and the outer cylinder has radius r_b = 12.0 mm. A power supply connected between the cylinders maintains a constant electric potential in the region $r_a \le r \le r_b$ between the cylinders given by



$$V_r = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{r_a}$$

where λ is the linear charge density (charge per unit length) on the inner cylinder. Find the equation for the electric field, E_r, as a function of r between the cylinders. Remember to show your work.

Solutions: *The electric field is equal to the negative of the potential gradient:*

$$E_r = -\frac{\partial V_r}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{r_a}\right) = -\frac{\lambda}{2\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\ln\frac{r}{r_a}\right) = -\frac{\lambda}{2\pi\varepsilon_0}\frac{\partial}{\partial r}(\ln r - \ln r_a)$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\frac{\partial}{\partial r}(\ln r) = -\frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{r}\right) = \boxed{-\frac{\lambda}{2\pi\varepsilon_0 r}}$$

CONSTANTS AND USEFUL EQUATIONS

k = Coulomb constant = 8.99 x 10^9 N m^2 C^{-2}
ε_0 = permittivity of free space = 8.85 x 10^{-12} C^2 N^{-1} m^{-2}
e = fundamental charge = 1.602 x 10^{-19} C
m_e = mass of electron = 9.11 x 10^{-31} kg

$m = 10^{-3}$ $\quad \mu = 10^{-6}$ $\quad n = 10^{-9}$ $\quad p = 10^{-12}$

Surface area of a sphere: $A = 4\pi r^2$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Area of a circle: $A = \pi r^2$

Circumference of a circle: $C = 2\pi r$

$$x = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 \qquad v_x = v_{x0} + a_x t \qquad v_x^2 = v_{x0}^2 + 2a_x x \qquad \vec{F} = m\vec{a}$$

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q} \qquad \vec{E} = k\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\Phi_E = \oint \vec{E}\bullet d\vec{A} = \oint E\cos\phi\, dA = \frac{Q_{encl}}{\varepsilon_0} \qquad V = \frac{U}{q} \qquad U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$V = k\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad W = qV_{ab} \qquad V_{ab} \equiv V_a - V_b = \int_a^b \vec{E}\bullet d\vec{l}$$

$$\vec{E} = -\vec{\nabla}V = -\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)V \qquad C = \frac{Q}{V_{ab}} \qquad C = \frac{\varepsilon_0 A}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \qquad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad C = C_1 + C_2 + C_3$$

$$u = \tfrac{1}{2}\varepsilon_0 E^2$$

$$\int\frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} \qquad \int\frac{dx}{\sqrt{a^2 + x^2}} = \ln\left(x + \sqrt{a^2 + x^2}\right)$$

$$\int\frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} \qquad \int\frac{dx}{\left(a^2 + x^2\right)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{a^2 + x^2}} \qquad \int\frac{x\,dx}{\left(a^2 + x^2\right)^{3/2}} = -\frac{1}{\sqrt{a^2 + x^2}}$$

☺☺☺ END OF TEST ☺☺☺

WJFW/2011-02-10