

NAME \_\_\_\_\_

(Do not write your ID number on this page; your ID number goes on all pages after this one.)

## PHYSICS 259 Midterm Test

February 17, 2011

90 minutes

***DO NOT TEAR OFF THIS PAGE!*** You may, however, tear off the last page.

This test consists of twelve (12) pages. Please ensure that you have all twelve pages.

**Do all questions.** Use of the Schulich-approved calculator is permitted. There is an equation sheet and table of integrals on the back of the final page.

***All answers for marking must be written on the ANSWER PAGES provided.*** Rough work may be written on this question paper, and will not be marked.

Part A of the exam consists of 10 multiple-choice questions, worth one mark each. **We recommend that you take no more than 30 minutes for this part of the exam.** Many of the questions require little or no calculations. Read each question carefully. If you get stuck ... move on. There is only one answer which will be accepted as correct for each question.

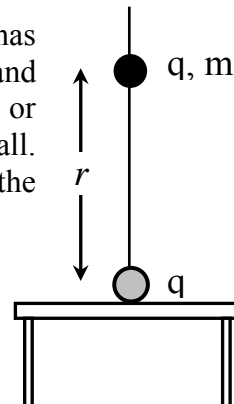
Part B of the exam consists of four “long-answer” questions worth a total of 21 marks. **We recommend that you take at least one hour for this part of the exam.** For full marks, ***all work must be shown and all reasoning clearly explained.*** Show all equations in symbols first, defining symbols. Where numbers are used, show how numbers are substituted into equations using correct units.

Assume all numbers are accurate to three significant figures unless more figures are specified.

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**PART A.** Multiple-choice questions. For each question, **mark your choice for the correct answer on the first page of the ANSWER SHEETS.** One mark each.

1. In the figure at the right, a small, grey ball is fixed to a tabletop and has been given a charge  $q$ . A vertical thread runs upward from this ball, and a black ball of mass  $m = 2.04$  g is free to slide without friction up or down this thread. The black ball has the same charge,  $q$ , as the grey ball. If the equilibrium distance,  $r$ , between the balls is 0.500 m, what is the value of  $q$ ? Choose the closest answer.



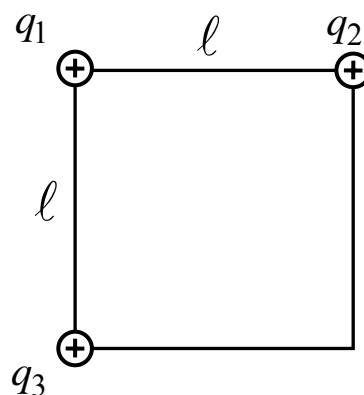
- a.  $1.80 \mu\text{C}$ .  
 b.  $4.24 \mu\text{C}$ .  
 c.  $1.06 \mu\text{C}$ .  
 d.  $0.746 \mu\text{C}$ .  
 e.  $2.12 \mu\text{C}$ .
2. A positively-charged, insulating rod is brought close to (but not touching) one end of a second rod that is hanging from a string. The charged rod attracts the rod on the string. From this observation we can be absolutely certain that the rod hanging from the string

- a. is a conducting rod.  
 b. is an insulating rod.  
 c. has a net charge.  
 d. does not have a net charge.  
 e. We are not certain of any of a, b, c or d.



3. Charges  $q_1 = q_2 = q_3 = 8.00 \mu\text{C}$  are at the corners of a square of sides  $\ell = 0.200$  m, as shown at the right. What is the magnitude of the force on  $q_1$  by the other two charges?

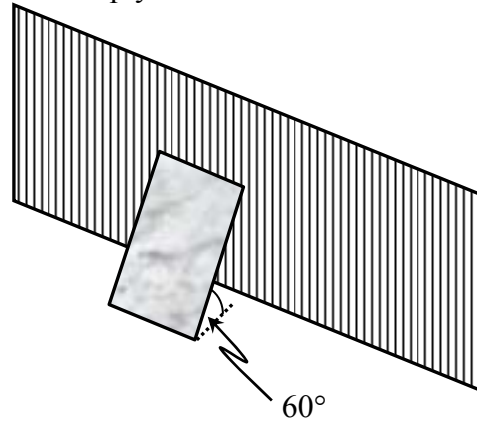
- a. 14.4 N.  
 b. 20.3 N.  
 c. 28.8 N.  
 d. 4.07 N.  
 e. 5.75 N.



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4. A sheet of plywood of area  $3.00 \text{ m}^2$  is leaning against the wall of a shed. The angle between the plywood and the ground is  $60^\circ$ . If there is an electric field of  $150 \text{ N/C}$  directed downward, what is the electric flux through the sheet of plywood? Choose the closest answer.

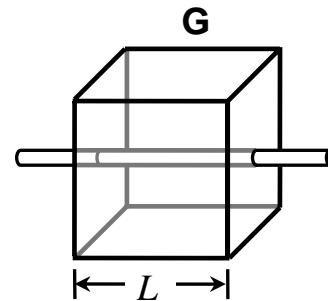
- a.  $450 \text{ N m}^2 \text{ C}^{-1}$
- b.  $390 \text{ N m}^2 \text{ C}^{-1}$
- c.  $225 \text{ N m}^2 \text{ C}^{-1}$
- d. 0.
- e.  $900 \text{ N m}^2 \text{ C}^{-1}$



5. In the diagram below, a horizontal, insulating rod of length  $2L$  carries a uniform, positive linear charge density (charge per unit length)  $\lambda$ . A student has drawn a Gaussian surface, labelled **G**, in the shape of a cube with edges of length  $L$ , centred on the rod.

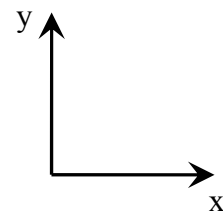
The net outward flux through this closed Gaussian surface is

- a.  $2L\lambda / \epsilon_0$ .
- b. impossible to calculate.
- c.  $L\lambda / \epsilon_0$ .
- d.  $6L^2\lambda / \epsilon_0$ .
- e.  $4L^2\lambda / \epsilon_0$ .



6. The electric potential in a particular area is given by  $V(x, y) = -2xy + y^2$ , where  $V$  is in volts,  $x$  and  $y$  are in metres, and the  $x$  and  $y$  axes are directed as indicated below. What is the direction of the electric field at the point  $x = 1.0 \text{ m}$ ,  $y = 3.0 \text{ m}$ ?

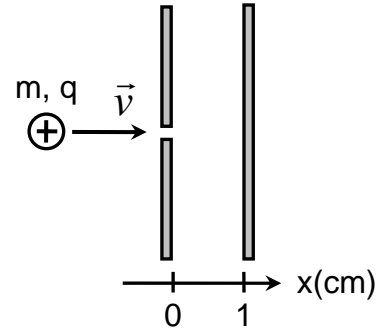
- a. Toward the upper right.
- b. Toward the lower right.
- c. Toward the upper left.
- d. Toward the lower left.
- e. Vertically downward.



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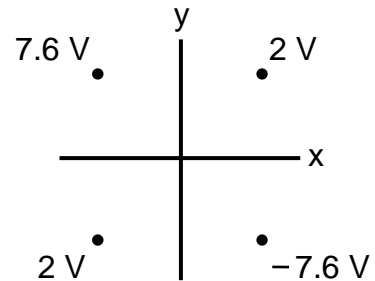
7. The figure below shows a side view of two metal plates 1.00 cm apart. The plates are large enough that the electric field between them is uniform. The potential difference between the plates is 80.0 V, with the right-hand plate being at the more positive potential. A proton (mass  $m = 1.67 \times 10^{-27}$  kg, charge  $q = +e$ ) travels through a small hole in the left-hand plate at a speed  $v = 1.00 \times 10^5$  m/s. What happens to the proton? Select the best answer.

- a. It turns back at  $x = 0.65$  cm.
- b. It *just* reaches the other plate ( $v = 0$  at  $x = 1.0$  cm).
- c. It turns back immediately ( $x = 0$ ) because it cannot enter a region of opposing electric field.
- d. It turns back at  $x = 0.35$  cm.
- e. It reaches the other plate at a speed  $v = 1.6 \times 10^5$  m/s.

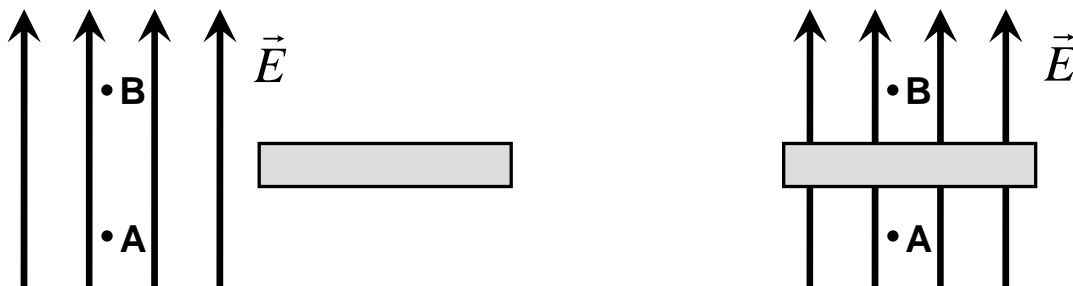


8. A person has measured the electric potential at four locations in the  $(x, y)$  plane (see figure below). Assuming that the  $z$ -component of the electric field is zero, what is the direction of the electric field relative to the axes shown?

- a. Toward the upper right.
- b. Toward the lower left.
- c. Toward the upper left.
- d. Toward the lower right.
- e. The electric field is zero.



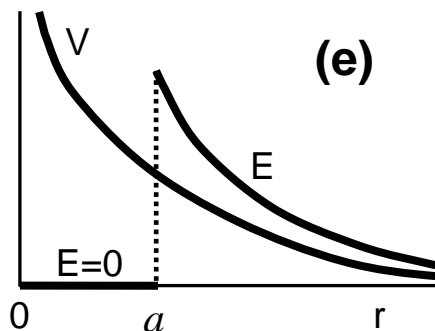
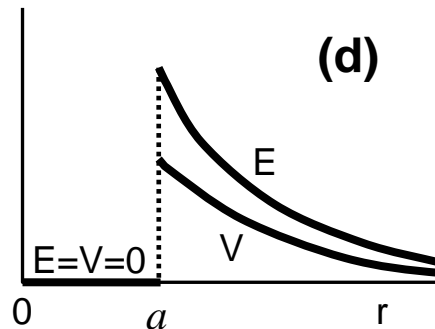
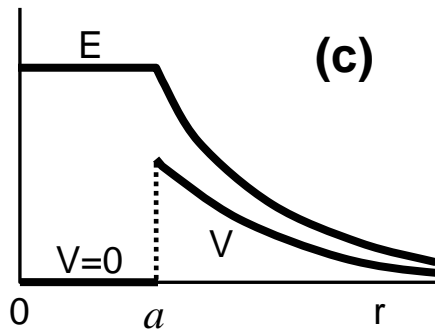
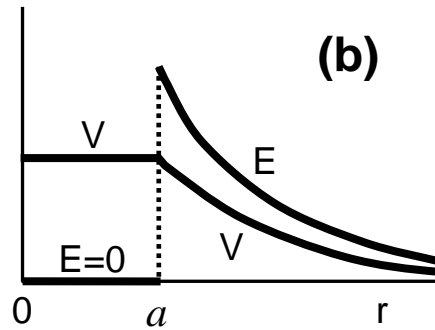
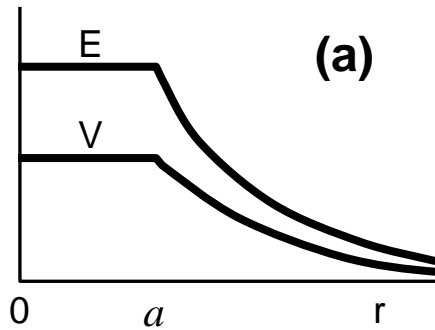
9. In the left-hand figure, below, points A and B are located in a region of uniform electric field,  $\vec{E}$ . An uncharged, thick, conducting plate (grey rectangle) is now slid between points A and B, as shown in the right-hand figure, while the electric field strength,  $E$ , is maintained at a constant value. What happens to the potential difference,  $V_{AB}$ , between points A and B, and which surface of the conducting plate becomes positively charged?



- a.  $V_{AB}$  decreases and the top surface of the plate becomes positively charged.
- b.  $V_{AB}$  decreases and the bottom surface of the plate becomes positively charged.
- c.  $V_{AB}$  remains the same and the top surface of the plate becomes positively charged.
- d.  $V_{AB}$  remains the same and the bottom surface of the plate becomes positively charged.
- e.  $V_{AB}$  increases and the bottom surface of the plate becomes positively charged.

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10. A solid metal sphere of radius  $a$  has been given an excess positive charge. Which of the five graphs, below, shows the behaviour of the electric potential,  $V$  (relative to  $V = 0$  at infinity), and the electric field strength,  $E$ , inside and outside this sphere as a function of the distance,  $r$ , from the centre of the sphere?

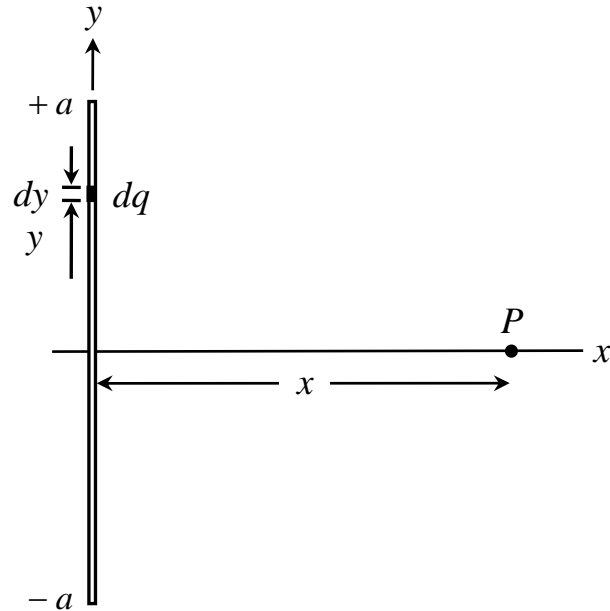


11. This is **Version A** of the exam. Please select (A) in the row marked "Version" in the Multiple Choice Answers area on the first Answer Sheet. *Be sure to do this now, before moving on to other questions on the paper.*

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**PART B.** Long-answer questions. Do all questions. Rough work may be done on this question paper, and will not be marked. *Finished solutions must be shown on the ANSWER PAGES. For full marks, show all work clearly and explain all reasoning.*

12. In the figure at the right, a thin, insulating rod of length  $2a$  lies along the  $y$ -axis with its centre at  $y = 0$ . Positive charge is distributed uniformly along the rod with a linear charge density (charge per unit length)  $\lambda$ .



- 1 mark a. How much charge,  $dq$ , is contained within any infinitesimally-short length,  $dy$ , of the rod? (See the figure at the right.) Express your answer in terms of  $dy$  and any other necessary quantities.
- 2 marks b. Find the  $x$ -component of the electric field vector that the small charge,  $dq$ , creates at point  $P$  in the figure above. Express your answer in terms of  $x$ ,  $y$ ,  $\lambda$ , and constants.
- 1 mark c. What is the total  $y$ -component,  $E_y$ , of the electric field vector at point  $P$  due to all of the charge on the rod shown above? Explain your answer in two sentences or less.
- 2 marks d. Find the magnitude of the total electric field,  $E$ , at point  $P$  due to all of the charge on the rod shown above. Express your answer in terms of  $a$ ,  $x$ ,  $\lambda$ , and constants.

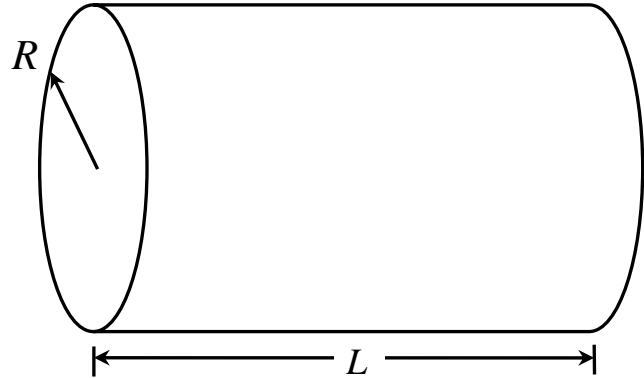
*The space below and on pages 7 and 11 is for ROUGH WORK which will not be marked. If you need more space, use the backs of the question sheets. Enter your finished solutions in the ANSWER PAGES. Work must be shown on the ANSWER PAGES for full marks.*

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***Space for rough work:***

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13. A very long, solid, insulating cylinder of radius  $R$  contains a uniform volume charge density  $\rho$ . The figure at the right shows a segment of length  $L$ , but the actual cylinder extends to large distances toward the left and right.



1 mark

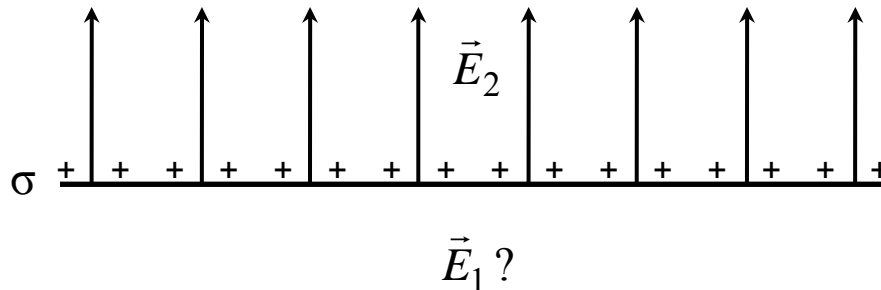
a. What total charge,  $Q$ , is contained in this segment of the cylinder? Express your answer in terms of quantities given in the problem, and constants.

2 marks

b. **Write down Gauss's law** and use it to derive the magnitude of the electric field,  $E$ , as a function of the radial coordinate,  $r$ , inside the long cylinder; i.e., in the region  $0 \leq r \leq R$ . Show your steps and reasoning, and make a drawing of the Gaussian surface you are using. Express your answer in terms of quantities given in the problem, and constants.

2 marks

c. (*Part c is not related to parts a or b.*) The horizontal line in the figure below represents a very large, very thin, horizontal sheet that carries a total charge per unit area  $\sigma = 2.66 \times 10^{-9} \text{ C/m}^2$ . (There may be other charges outside the figure that are not shown.) The electric field in the space above the sheet is  $E_2 = 1200 \text{ N/C}$ . What are the magnitude and direction of the electric field,  $\vec{E}_1$ , in the space below the sheet?



**Enter your finished solutions in the ANSWER PAGES. You must show your work to receive full marks.**

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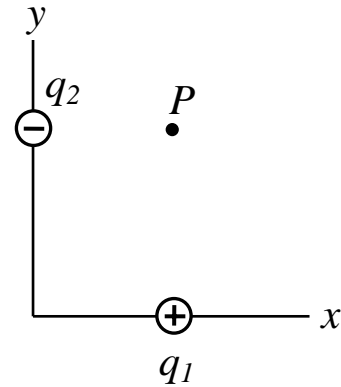
14. In the figure at the right, a point charge  $q_1 = +5.00 \text{ nC}$  is located on the  $x$ -axis at  $x = 3.00 \text{ cm}$ , and a second point charge  $q_2 = -3.00 \text{ nC}$  is located on the  $y$ -axis at  $y = 4.00 \text{ cm}$ .

2 marks

a. Find the electric potential,  $V$ , at point  $P$  at the position  $(x,y) = (3.00 \text{ cm}, 4.00 \text{ cm})$ . Assume  $V = 0$  at infinity.

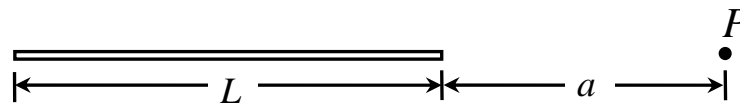
1 mark

b. A third charge,  $q_3 = +2.00 \text{ nC}$ , is now placed at point  $P$ . What is the potential energy of  $q_3$  relative to the other two charges?



2 marks

c. (*Part c is not related to parts a or b.*) The figure below shows a thin, insulating rod of length  $L$ . Positive charge is distributed uniformly along the rod with a linear charge density (charge per unit length)  $\lambda$ . Find the electric potential,  $V$ , at point  $P$  on the axis of the rod at a distance  $a$  from the end of the rod, as shown. Express your answer in terms of  $L$ ,  $a$ ,  $\lambda$  and constants.



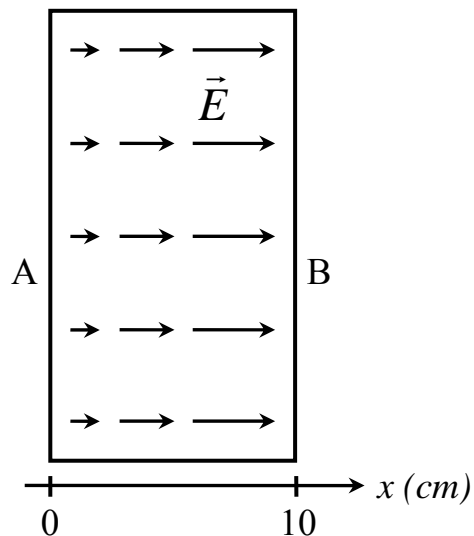
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15. The figure at the right shows a thick slab of insulating material of thickness  $L = 10.0$  cm, with its left side at  $x = 0$ . The electric field inside the slab is given by

$$\vec{E} = Cx^2 \hat{i}$$

where  $C$  is a constant. The potential difference between sides A and B is 75.0 V.



2 marks

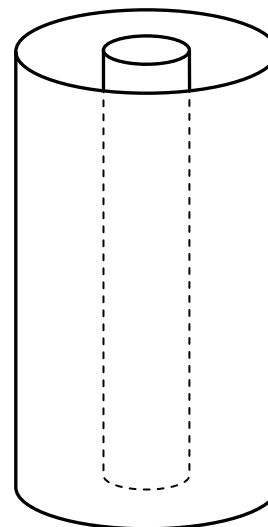
a. Find the numerical value of the constant,  $C$ , with correct units. [HINT: How are electric field and potential difference related?]

1 mark

b. Which side of the block (side A or side B) is at the higher potential?

2 marks

c. (Parts c and d are not related to parts a and b.) The two metal cylinders in the figure at the right are co-axial and are separated by vacuum. The inner cylinder has radius  $r_a = 3.00$  mm and the outer cylinder has radius  $r_b = 12.0$  mm. A power supply connected between the cylinders maintains a constant electric potential in the region  $r_a \leq r \leq r_b$  between the cylinders given by



$$V_r = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r}{r_a}$$

where  $\lambda$  is the linear charge density (charge per unit length) on the inner cylinder. Find the equation for the electric field,  $E_r$ , as a function of  $r$  between the cylinders. Remember to show your work.

**Enter your finished solutions in the ANSWER PAGES. You must show your work to receive full marks.**

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***Space for rough work:***

## *CONSTANTS AND USEFUL EQUATIONS*

$$k = \text{Coulomb constant} = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\epsilon_0 = \text{permittivity of free space} = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$e = \text{fundamental charge} = 1.602 \times 10^{-19} \text{ C}$$

$$m_e = \text{mass of electron} = 9.11 \times 10^{-31} \text{ kg}$$

$$m = 10^{-3} \quad \mu = 10^{-6} \quad n = 10^{-9} \quad p = 10^{-12}$$

$$\text{Surface area of a sphere: } A = 4\pi r^2$$

$$\text{Volume of a sphere: } V = \frac{4}{3}\pi r^3$$

$$\text{Area of a circle: } A = \pi r^2$$

$$\text{Circumference of a circle: } C = 2\pi r$$

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_{x0}t + \frac{1}{2}\mathbf{a}_x t^2$$

$$\mathbf{v}_x = \mathbf{v}_{x0} + \mathbf{a}_x t$$

$$v_x^2 = v_{x0}^2 + 2a_x x$$

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{F}} = k \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}$$

$$\vec{\mathbf{E}} = k \frac{q}{r^2} \hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E \cos\phi dA = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$V = \frac{U}{q}$$

$$U = k \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$V = k \frac{q}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$W = qV_{\text{ab}}$$

$$V_{\text{ab}} \equiv V_a - V_b = \int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

$$\vec{\mathbf{E}} = -\vec{\nabla}V = -\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)V$$

$$C = \frac{Q}{V_{\text{ab}}}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C = C_1 + C_2 + C_3$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln\left(x + \sqrt{a^2 + x^2}\right)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a}$$

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{a^2 + x^2}}$$

$$\int \frac{x dx}{(a^2 + x^2)^{3/2}} = -\frac{1}{\sqrt{a^2 + x^2}}$$

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