Solutions - Midterm #2

Multiple Choice Section Questions (1-4)

Question 1 *Find* $f'(\ln(2))$ *when* $f(x) = xe^{3x}$

A) $8 + 24\ln(2)$ **B)** $8 + 3\ln(2)$ **C)** $24 + 3\ln(2)$ **D)** $1 + 2\ln(2)$ **E)** $-3\ln(2)$

$f'(x) = 3xe^{3x} + e^{3x}$

$f'(\ln(2)) = 3\ln(2)e^{3\ln(2)} + e^{3\ln(2)}$

$= 24\ln(2) + 8$

Question 2 *Suppose that the demand function for a product is given by* $p = -x^2 - 2x + 20$. *What is the elasticity of demand when* $x = 3$? *Is demand elastic or inelastic?*

A) $\eta = -\frac{1}{5}$, elastic **B)** $\eta = -\frac{1}{5}$, inelastic **C)** $\eta = -\frac{5}{24}$, elastic

D) $\eta = -\frac{5}{24}$, inelastic **E)** $\eta = -\frac{6}{11}$, elastic

$x = 3 \Rightarrow p = 5$

$\frac{\partial p}{\partial x} = -2x - 2 = -8$

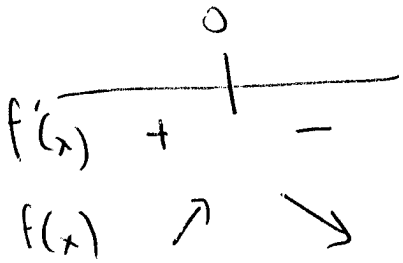
$\eta = \frac{5/3}{-8} = \frac{-5}{24}$

Question 3 *Given the function* $f(x) = \frac{1}{x^2+1}$, *which of the following statements is correct? (Only one is correct.)*

A) $f(x)$ has a local minimum at $x = -2$
B) $f(x)$ has a local maximum at $x = -2$
C) $f(x)$ has an inflection point at $x = -2$
D) $f(x)$ has a local maximum at $x = 0$
E) $f(x)$ has a local minimum at $x = 0$

$f(x) = (x^2+1)^{-1}$

$f'(x) = \frac{-2x}{(x^2+1)^2}$ One CP at 0



Question 4 *Consider the function* $g(x) = x^4 + 2x^3 - 12x^2 - 6x + 2$. *On what interval or intervals is the function concave down?*

A) $(-3, 2)$ **B)** $(2, \infty)$ **C)** $(-2, 1)$ **D)** $(-\infty, -2)$ **E)** $(0, 1)$

$g'(x) = 4x^3 + 6x^2 - 24x - 6$

$g''(x) = 12x^2 + 12x - 24 = 12(x^2 + x - 2)$

$= 12(x+2)(x-1)$

Possible IPs at $-2, 1$

Long Answer Section Questions (5-7)

Question 5 (14 points) *A population of zombies is growing exponentially. In 1999, there were 12,000 zombies, and in 2005 there were 15,000 zombies.*

(a) *(4 points) Find a formula which describes the size of the zombie population as a function of time (measured in years).*

(b) *(4 points) What will the population size be in 2024?*

(c) *(6 points) How many years will it take before there are ~~22,000~~ 28,000 zombies on the island?*

a) $P(t) = P_0 b^t = 12{,}000\, b^t$

$P(6) = 15{,}000 \Rightarrow 15{,}000 = 12{,}000\, b^6$

$\Rightarrow \frac{5}{4} = b^6 \Rightarrow b = \left(\frac{5}{4}\right)^{1/6}$

So $P(t) = 12{,}000 \left(\frac{5}{4}\right)^{t/6}$

b) $P(25) = 12{,}000 \left(\frac{5}{4}\right)^{25/6}$

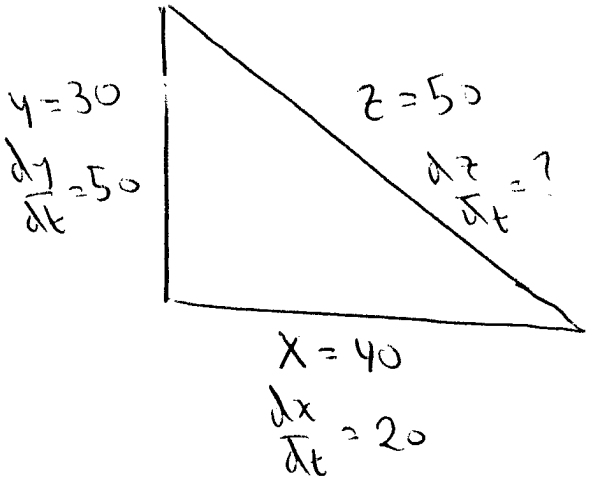
c) Solve $28{,}000 = 12{,}000 \left(\frac{5}{4}\right)^{t/6}$ for t.

$\frac{7}{3} = \left(\frac{5}{4}\right)^{t/6} \Rightarrow \ln\left(\frac{7}{3}\right) = \frac{t}{6} \ln\left(\frac{5}{4}\right)$

$\Rightarrow t = \dfrac{6 \ln\left(\frac{7}{3}\right)}{\ln\left(\frac{5}{4}\right)}$

Question 6 (14 points)

A car is 40 miles east of an intersection and is travelling east away from the intersection at a rate of 20 miles per hour. Another car is 30 miles north of the intersection and is travelling north away from the intersection at a rate of 50 miles per hour. At what rate is the distance between them changing? **Be sure to draw a picture to help with your answer.**



$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(40)(20) + (30)(50) = 50\left(\frac{dz}{dt}\right)$

$2300 = 50\frac{dz}{dt}$

$\frac{dz}{dt} = 46$

Question 7 (14 points) *If a tour company charges 80 dollars per person for its tour, they get 300 customers. For each 5 dollar decrease in price, an additional 30 customers are attracted. At what price will the company maximize revenue?* **Be sure to explain why your answer is an absolute maximum.**

P	x
80	300
75	330

$m = \frac{\Delta P}{\Delta x} = -\frac{1}{6}$

$y = -\frac{1}{6}x + b$

$80 = -\frac{1}{6}(300) + b \Rightarrow b = 130$

$D(x) = -\frac{1}{6}x^2 + 130x$

$D'(x) = -\frac{1}{3}x + 130 \Rightarrow$ 1 CP at $x = 390$

Price is 65 $

Absolute max, since $D(x)$ is a CD parab.

Bonus Question (a half point): Name the film star whose first starring role was in the 1958 film *The Blob.*

Multiple Choice Section Questions (1-4)

Question 1 *Find* $f'(\ln(3))$ *when* $f(x) = xe^{2x}$

A) $2 + 3\ln(2)$ **B)** $-3 + 2\ln(3)$ **C)** $9 + 18\ln(3)$ **D)** $18 + 9\ln(2)$ **E)** $-3\ln(2)$

$f'(x) = e^{2x} + 2xe^{2x}$

$f'(\ln(3)) = e^{2\ln(3)} + 2\ln(3)\, e^{2\ln(3)}$

$= e^{\ln(9)} + 2\ln(3)\, e^{\ln(9)}$

$= 9 + 18\ln(3)$

Question 2 *Suppose that the demand function for a product is given by* $p = -x^2 - 3x + 15$. *What is the elasticity of demand when* $x = 2$? *Is demand elastic or inelastic?*

A) $\eta = -\frac{3}{20}$, elastic **B)** $\eta = -\frac{1}{2}$, inelastic **C)** $\eta = -\frac{5}{14}$, elastic

D) $\eta = -\frac{1}{3}$, elastic **E)** $\eta = -\frac{5}{14}$, inelastic

$x = 2 \Rightarrow p = 5$

$p' = -2x - 3$

$p'(2) = -7$

$\eta = \frac{p/x}{dp/dx} = \frac{\frac{5}{2}}{-7} = -\frac{5}{14}$ · inelastic

Question 3 *Given the function* $f(x) = \frac{-1}{x^2+4}$, *which of the following statements is correct? (Only one is correct.)*

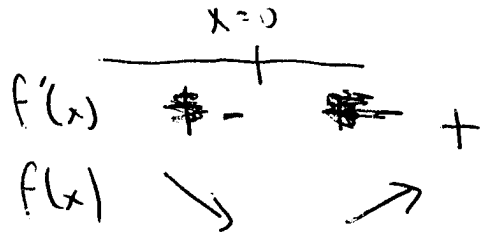
A) $f(x)$ has a local minimum at $x = 0$
B) $f(x)$ has a local maximum at $x = 0$
C) $f(x)$ has an inflection point at $x = 2$
D) $f(x)$ has a local maximum at $x = -2$
E) $f(x)$ has a local minimum at $x = 2$

$f(x) = (x^2+4)^{-1}$

$f'(x) = +(x^2+4)^{-2}(2x)$

$= \frac{+2x}{(x^2+4)^2}$

One CP at $x = 0$



It's a local min.

Question 4 *Consider the function* $g(x) = x^4 - 2x^3 - 12x^2 + 8x - 1$. *On what interval or intervals is the function concave down?*

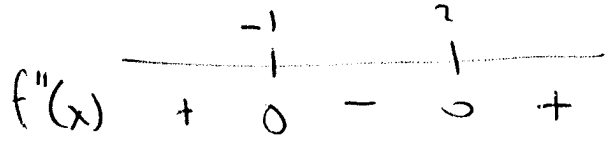
A) $(2, \infty)$ **B)** $(-1, 2)$ **C)** $(-3, 2)$ **D)** $(-\infty, 1)$ **E)** $(0, 2)$

$g'(x) = 4x^3 - 6x^2 - 24x + 8$

$g''(x) = 12x^2 - 12x - 24$

$= 12(x^2 - x - 2)$

$= 12(x-2)(x+1)$



2 possible IP's at $x = -1, 2$

Long Answer Section Questions (5-7)

Question 5 (14 points) *A population of zombies is growing exponentially. In 1999, there were 8,000 zombies, and in 2005 there were 12,000 zombies.*

(a) *(4 points) Find a formula which describes the size of the zombie population as a function of time (measured in years).*

(b) *(4 points) What will the population size be in 2024?*

(c) *(6 points) How many years will it take before there are 18,000 zombies on the island?* 28,000

a) $P(t) = P_0 b^t = 8{,}000\, b^t$

$P(6) = 12{,}000 \Rightarrow 12{,}000 = 8{,}000\, b^6$

$\Rightarrow \left(\frac{3}{2}\right) = b^6 \Rightarrow b = \left(\frac{3}{2}\right)^{1/6}$

So $P(t) = 8{,}000 \left(\frac{3}{2}\right)^{t/6}$

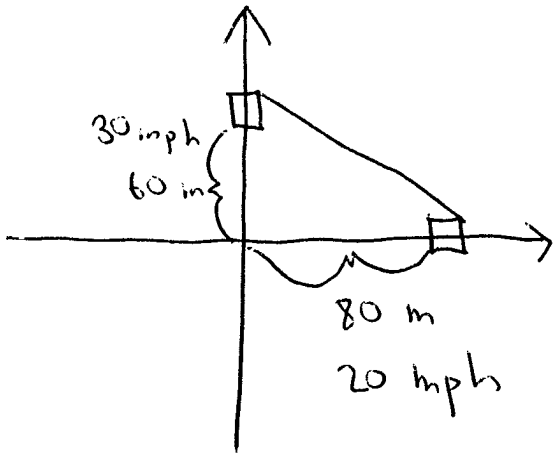
b) $P(25) = 8{,}000 \left(\frac{3}{2}\right)^{25/6}$

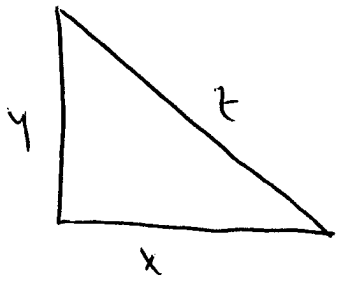
c) Solve $28{,}000 = 8{,}000 \left(\frac{3}{2}\right)^{t/6}$ for t

$\Rightarrow \frac{7}{2} = \left(\frac{3}{2}\right)^{t/6} \Rightarrow \ln\left(\frac{7}{2}\right) = \frac{t}{6} \ln\left(\frac{3}{2}\right)$

$\Rightarrow t = \dfrac{6 \ln\left(\frac{7}{2}\right)}{\ln\left(\frac{3}{2}\right)}$

Question 6 (14 points) *A car is 80 miles east of an intersection and is travelling east away from the intersection at a rate of 20 miles per hour. Another car is 60 miles north of the intersection and is travelling north away from the intersection at a rate of 30 miles per hour. At what rate is the distance between them changing?* **Be sure to draw a picture to help with your answer.**





$z = \sqrt{60^2 + 80^2} = 100$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$ $\qquad \frac{dx}{dt} = 34$

$80(20) + 60(30) = 100\frac{dz}{dt}$

$1600 + 1800 = 100\frac{dz}{dt}$

$3400 = 100\frac{dz}{dt}$

Question 7 (14 points) *If a tour company charges 105 dollars per person for its tour, they get 600 customers. For each 5 dollar decrease in price, an additional 40 customers are attracted. At what price will the company maximize revenue?* **Be sure to explain why your answer is an absolute maximum.**

P	x
105	600
100	640

$m = \frac{\Delta P}{\Delta x} = \frac{-1}{8}$

$y = \frac{-1}{8}x + b$

$105 = -\frac{1}{8}(600) + b$

$105 = -75 + b$

$b = 180$

$y = -\frac{1}{8}x + 180$

$R(x) = -\frac{1}{8}x^2 + 180x$

$R'(x) = -\frac{1}{4}x + 180 \Rightarrow x = 720$ is a CP

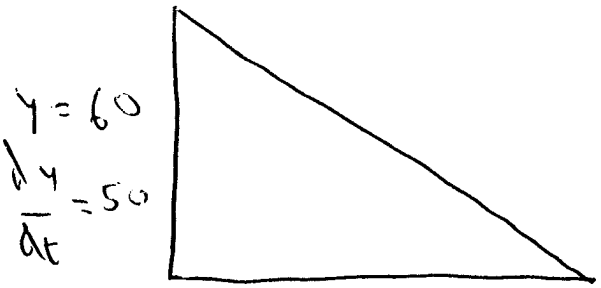
Corresponding price is 90 $

It is an absolute max, since $R(x)$ is a CD parabola.

Bonus Question (a half point): Name the film star whose first starring role was in the 1958 film *The Blob.*

Steve McQueen

Question 6 (14 points) *A car is 80 miles east of an intersection and is travelling east away from the intersection at a rate of 10 miles per hour. Another car is 60 miles north of the intersection and is travelling north away from the intersection at a rate of 50 miles per hour. At what rate is the distance between them changing?* **Be sure to draw a picture to help with your answer.**



$z = \sqrt{60^2 + 80^2}$

$= 100$

$\frac{dz}{dt} = ?$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$80(10) + 60(50) = 100\frac{dz}{dt}$

$800 + 3000 = 100\frac{dz}{dt}$

$\frac{dz}{dt} = 38$

Question 7 (14 points) *If a tour company charges 95 dollars per person for its tour, they get 600 customers. For each 5 dollar decrease in price, an additional 40 customers are attracted. At what price will the company maximize revenue?* **Be sure to explain why your answer is an absolute maximum.**

P	x
95	600
90	640

$m = \frac{\Delta P}{\Delta x} = \frac{-1}{8}$

$y = -\frac{1}{8}x + b$

$95 = -\frac{1}{8}(600) + b$

$95 = -75 + b$

$b = 170$

$y = -\frac{1}{8}x + 170$

$R(x) = -\frac{1}{8}x^2 + 170$

$R'(x) = -\frac{1}{4}x + 170$

One CP at $x = 680$

Corresponding price is 85 dollars

Absolute max, since $R(x)$ is a CD parabola.

Bonus Question (a half point): Name the film star whose first starring role was in the 1958 film *The Blob*.