MATH 1300-MIDTERM # 1-2010

Version C.

NAME and I.D.#_____

INSTRUCTIONS: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

For long answer questions, YOU MUST SHOW YOUR WORK

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Multiple Choice Answers:

#1	#2	#3	#4
A	B	B	C

Multiple Choice Questions (1-4)

Question 1 *Find the limit*
$\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$.

A) 6 **B)** 12 **C)** 9 **D)** -8 **E)** 3

$$\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} = \lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x\to 9} (\sqrt{x}+3) = 6$$

Question 2 *For what value of a is the following function continuous?*

$$f(x) = \begin{cases} x^2 + ax - 6 & \text{if } x > 2 \\ \frac{8}{x+2} & \text{if } x \le 2. \end{cases}$$

A) $-\frac{1}{4}$ **B)** 2 **C)** 4 **D)** $\frac{8}{3}$ **E)** -6

$$2^2 + a\cdot 2 - 6 = \frac{8}{2+2}$$

$$-2 + 2a = 2,$$

$$a = 2$$

Question 3 *Find the equation of the tangent line of the function*

$f(x) = \frac{x^2}{3-2x}$ *at* $x = 1$.

A) $y = \frac{1}{2}x + \frac{1}{2}$ **B)** $y = 4x - 3$ **C)** $y = -x + 2$ **D)** $y = \frac{1}{3}x - 3$ **E)** $y = 4x - 7$

$x_0 = 1, \quad y_0 = 1$

$$f'(x) = \frac{2x(3-2x) - x^2(-2)}{(3-2x)^2} = \frac{6x - 4x^2 + 2x^2}{(3-2x)^2} = \frac{6x - 2x^2}{(3-2x)^2}$$

$f'(1) = 4$

$y = 4(x-1) + 1 = 4x - 3$

Question 4 *Let* $f(x) = \sqrt{8-2x}$. *Find* $f^{-1}(2)$, *where* $f^{-1}(x)$ *is the inverse of* $f(x)$.

A) $\sqrt{2}$ **B)** -4 **C)** 2 **D)** $\sqrt{8}$ **E)** 0

$x = \sqrt{8-2y}$

$x^2 = 8 - 2y,$

$y = \frac{8-x^2}{2} = f^{-1}(x)$

$f^{-1}(2) = \frac{8-4}{2} = 2$

Long Answer Questions (5-7)

Question 5 (14 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \frac{1}{x-3}.$$

Solution

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left(\frac{1}{x+\Delta x - 3} - \frac{1}{x-3}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(\frac{1}{x+\Delta x - 3} - \frac{1}{x-3}\right) \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x-3) - (x+\Delta x - 3)}{(x+\Delta x - 3)(x-3)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{(x+\Delta x - 3)(x-3)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x - 3)(x-3)} = \frac{-1}{(x-3)^2} = f'(x), \quad x \neq 3.$$

Question 6 (14 points)

Suppose that a deposit of P dollars is invested for t years into a bank that gives 4% annual interest.

a) Assume that interest is compounded 2 times per year. Let P be the initial deposit. Suppose that after 6 years, there is 10,000 dollars in the account. Find the initial deposit P. **Do not simplify your answer.**

b) Now assuming that the 4% annual interest is compounded continuously, find the time needed for an initial deposit of 3,000 dollars to triple. **Do not simplify your answer.**

Solution

a) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$r = 0.04, \quad n = 2, \quad t = 6, \quad A = 10\,000$

$10\,000 = P \cdot \left(1 + \frac{0.04}{2}\right)^{2 \cdot 6}$

$10\,000 = P \cdot (1.02)^{12}$

$P = 10\,000 \cdot (1.02)^{-12}$

b) $A = P e^{rt}$

$P = 3\,000, \quad A = 9\,000, \quad r = 0.04$

$9\,000 = 3\,000\, e^{0.04 \cdot t}$

$3 = e^{0.04 \cdot t}$

$\ln 3 = 0.04 \cdot t$

$t = \frac{\ln 3}{0.04}$

Question 7 (12 points)

The function $y = f(x)$ is defined implicitly by

$$6\sqrt{xy} - x = 5y.$$

Find the equation of the tangent line to the graph of $y = f(x)$ at $(2, 2)$.

Solution

- $\left(6(xy)^{1/2} - x\right)' = (5y)'$

$$6\,\frac{1}{2}(xy)^{-1/2}(xy)' - 1 = 5y'$$

$$3\,\frac{1}{\sqrt{x\cdot y}}(y + xy') - 1 = 5y'$$

- $3\,\frac{1}{\sqrt{4}}(2 + 2y') - 1 = 5y'$

$$3 + 3y' - 1 = 5y'$$

$$y' = f'(2) = 1$$

- $y = f(2) + f'(2)(x - 2)$

$$= 2 + 1\cdot(x - 2)$$

$$y = x$$