Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MT1320D Calculus 1 **Final Exam**

Professor: José Malagón-López 16 April 2011

NAME: ____________________

STUDENT NUMBER: ____________________

- No calculators or other electronic aids allowed.
- No notes, books or other papers allowed.
- Answer all questions in the space provided. You must justify your answers and explain your reasoning.
- Do not unstaple the booklet.
- There are 17 pages, including this one. There are 17 questions worth a total of 100 marks.
- Answer all questions in the space provided. If necessary, you may continue your answer on the reverse side or in the extra sheet provided at the end. Indicate clearly that you have done so.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	total
/4	/3	/3	/5	/3	/5	/5	/5	/5	/4	/3	/3	/3	/3	/6	/4	/5	/100

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1. Compute the derivative of the following functions.
YOU DO NOT NEED TO SIMPLIFY YOUR ANSWERS.

[3] a) $f(x) = \sqrt{x}\arctan(x)$.

[3] b) $f(x) = \dfrac{2^x - 1}{\ln(x)}$.

[4] c) $f(x) = \cos\left(x^{-1} - 5x^2 + 3\right)$.

[4] d) $f(x) = e^{\sin(3x^4-1)}$.

[**3**] 2. For $f(x) = \dfrac{2}{x}$, use the **limit definition** to compute the derivative $f'(x)$.

[**3**] 3. Given the function $f(x) = \tan(x)$, find the linear approximation of $f(x)$ at $a = 0$ and then estimate $f(0.15)$.

[5] 4. A curve on the plane is given by $x^2 + xy + y^2 - 5x = 4$. Find the slope of the tangent line on the curve at the point $(0, 2)$.

[3] 5. A cylindrical tank with radius 5 m is being filled with water at a rate of $3\,\mathrm{m}^3/\mathrm{min}$. How fast is the height of the water increasing?

[5] 6. A 24 m^2 rectangular garden is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimension for the outer rectangle will require the smallest total length of fence?

7. Compute the following limits.

[**4**] a) $\lim_{x\to\infty} \dfrac{\ln(x)}{2\sqrt{x}}$.

[**5**] b) $\lim_{x\to 0^+} \sin(x)\ln(x)$.

[**5**] 8. Find the domain, intercepts and asymptotes for $f(x) = \dfrac{2}{e^x + 1}$.

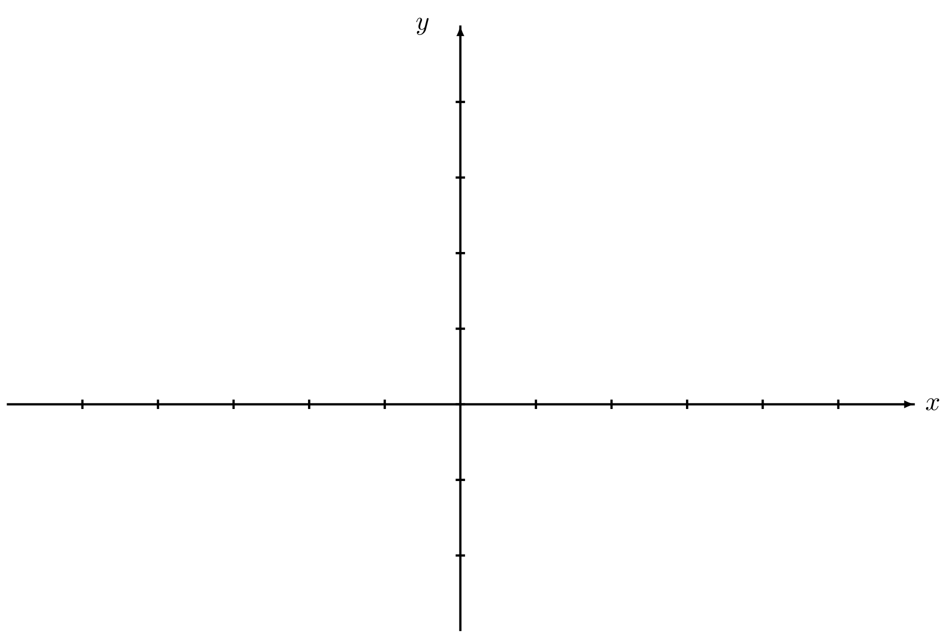
[**5**] 9. Find the intervals of increase and decrease and the local maximum and minimum for the function $f(x)$ whose **first derivative** is given by $f'(x) = \dfrac{(x-5)^2(x+1)}{x-1}$.

[4] 10. Sketch a graph of the function $y = f(x)$ with the following properties.

- Domain : $(-\infty, 1) \cup (1, \infty)$.
- y-intercept: $y = 2$.
- x-intercepts: $x = -1$.
- $\lim_{x \to 1} f(x) = \infty$.
- $\lim_{x \to -\infty} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = 2$.
-

Interval	f'
$(-\infty, 1)$	Positive
$(1, \infty)$	Negative

Interval	f''
$(-\infty, -1)$	Negative
$(-1, 1)$	Positive
$(1, \infty)$	Positive



[3] 11. Find $f(x)$ if $f'(x) = 8x + \sin(x)$ and $f(0) = \pi$.

[3] 12. Estimate the area under the graph of $f(x) = \sqrt{x}$ from 0 to 4 using four approximating rectangles and right endpoints.

[3] 13. If $f(x)$ and $g(x)$ are continuous functions such that

$$\int_2^0 f(x)\,dx = \pi, \quad \int_0^2 g(x)\,dx = 6\pi,$$

find the value of

$$\int_0^2 (3g(x) + f(x))\,dx.$$

[3] 14. Find $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \sqrt{1+t^3}\,dt.$

15. Find the following indefinite integrals using any appropriate integration technique.

[3] a) $\displaystyle\int \frac{4x^2 + x\sqrt{x}}{x^3}\,dx.$

[4] b) $\displaystyle\int xe^{x^2+1}\,dx.$

[4] c) $\int x^2 \ln x \, dx.$

[**6**] d) $\displaystyle\int \frac{9}{x(x^2+9)}\,dx.$

[6] e) $\displaystyle\int \frac{x^3}{\sqrt{16-x^2}}\,dx.$

[**4**] 16. Evaluate $\int_0^1 \frac{e^x+1}{e^x+x}\,dx$.

[5] 17. Find the following limit

$$\lim_{x \to 0} \frac{\int_0^x (e^t - 1 - t)\,dt}{x^3}.$$

This page is left blank intentionally. It may be used for any answer which you could not fit in the space provided.