

Limits

1. Calculating Limits Using A Graph

Use the graph of $y = f(x)$ given to answer the following questions. Exact values should be given, not approximations. Use the symbols ∞ , $-\infty$ and DNE appropriately.

a) $\lim_{x \rightarrow 4} f(x) = -3$

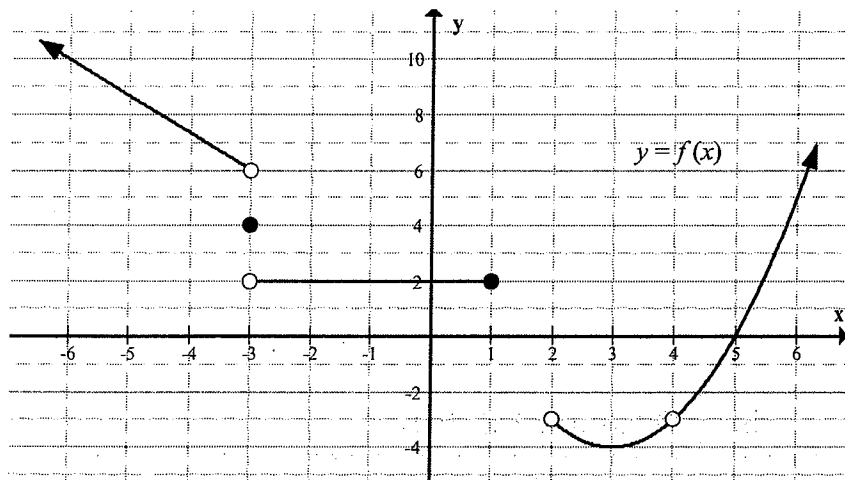
b) $\lim_{x \rightarrow 3} f(x) = \text{DNE}$

c) $\lim_{x \rightarrow 4} f(x) = \frac{22}{3}$

d) $\lim_{x \rightarrow 2^+} f(x) = -3$

e) $\lim_{x \rightarrow 2^-} f(x) = \text{DNE}$

f) $\lim_{x \rightarrow -e} f(x) = 2$



2. Calculating Limits Algebraically

Find each limit if it exists. Show all work necessary to justify your result. Use the symbols ∞ , $-\infty$ and DNE appropriately.

$$\text{a) } \lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{x - 4} = \lim_{x \rightarrow 4} \frac{(x-4)(x-2)}{x-4} = \lim_{x \rightarrow 4} (x-2) = 4-2 = 4$$

$$\begin{aligned} \text{b) } \lim_{x \rightarrow 25} \frac{x-25}{\sqrt{x}-5} &= \lim_{x \rightarrow 25} \frac{(x-25)(\sqrt{x}+5)}{(\sqrt{x}-5)(\sqrt{x}+5)} = \lim_{x \rightarrow 25} \frac{(x-25)(\sqrt{x}+5)}{x-25} \\ &= \lim_{x \rightarrow 25} (\sqrt{x}+5) = \sqrt{25} + 5 = 10 \end{aligned}$$

$$\text{c) } \lim_{x \rightarrow 9} \frac{\sqrt{x}-9}{\sqrt{x}-3} \begin{matrix} \rightarrow -6 \\ \rightarrow 0 \end{matrix} = \text{DNE} \quad \text{Note: } \lim_{x \rightarrow 9^+} \frac{\sqrt{x}-9}{\sqrt{x}-3} = -\infty \text{ and } \lim_{x \rightarrow 9^-} \frac{\sqrt{x}-9}{\sqrt{x}-3} = +\infty$$

$$\text{d) } \lim_{x \rightarrow 2^+} \frac{3x^2 - x + 1}{2 - x} \begin{matrix} \rightarrow \infty \\ \rightarrow 0^- \end{matrix} = -\infty$$

$$\text{e) } \lim_{x \rightarrow \infty} \frac{2x^3 + x^{-2} - 1}{7x - 9x^3} = \lim_{x \rightarrow \infty} \frac{(2x^3 + \frac{1}{x^2} - 1) \cdot \frac{1}{x^3}}{(\frac{7}{x^2} - 9x^3) \cdot \frac{1}{x^3}} = \lim_{x \rightarrow \infty} \frac{2 + \frac{1}{x^5} - \frac{1}{x^3}}{\frac{7}{x^2} - 9} = -\frac{2}{9}$$