

**MAT 2371**  
**Midterm Examination**

**October 21, 2010**  
**Time: 80 minutes**

**Professor G. Ivanoff**

**Student Number:** \_\_\_\_\_

**Family Name:** \_\_\_\_\_ **First Name:** \_\_\_\_\_

**This is an open book examination. Calculators are the only electronic device permitted permitted. Record your answer to each question in the table below.**

Question	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

**NOTE: At the end of the examination, hand in only this page. You may keep the questionnaire.**

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1. Suppose that  $A$ ,  $B$  and  $C$  are independent events. If  $P(A) = 0.5$ ,  $P(B) = 0.4$  and  $P(C) = 0.6$ , find  $P((A \cup B) \cap C)$ .
- (A) 0.60      (B) 0.58      (C) 0.12      (D) 0.68      (E) 0.42
2. Suppose that  $A$ ,  $B$  and  $C$  are mutually exclusive events. If  $P(A) = 0.5$ ,  $P(B) = 0.4$  and  $P(C) = 0.6$ , find  $P((A \cap B) \cup C)$ .
- (A) 0.60      (B) 0.58      (C) 0.12      (D) 0.68      (E) 0.42
3. Telephone numbers in the National Capital region consist of 10 digits. The first 3 digits are either 613 (Ontario telephone numbers) or 819 (Quebec telephone numbers); the 4th digit cannot be 0 or 1, the 5th digit cannot be 0, and there are no restrictions on the remaining 5 digits. How many different telephone numbers are possible in the National Capital region?
- (a)  $72 \times 10^5$     (b)  $72 \times 10^8$     (c)  $3! \times 72 \times 10^5$     (d)  $144 \times 10^5$     (e)  $10^5$
4. I purchase milk at three different grocery stores. The probability that milk purchased at store A will be fresh for at least two weeks is 0.7, and the corresponding probabilities for milk purchased at stores B and C are 0.4 and 0.3, respectively. Suppose that I do my shopping at store A 20% of the time, at store B 30% of the time and at store C 50% of the time. I bought a carton of milk today. What is the probability that it will be fresh for at least two weeks?
- (A) 0.500      (B) 0.700      (C) 0.410      (D) 0.140      (E) 0.120
5. Referring to the situation in problem #4, if in two weeks my milk is still fresh, what is the probability that I purchased it at store C?
- (A) 0.500      (B) 0.300      (C) 0.293      (D) 0.341      (E) 0.366

6. A student in this class is writing three midterms in math this week. The probability that he will pass MAT 2371 is 0.8, the probability that he will pass MAT 2120 is 0.7, and the probability that he will pass MAT 2141 is 0.6. If his results on the three exams are independent, what is the probability that he will fail at most one exam?

(A) 0.788      (B) 0.664      (C) 0.024      (D) 0.976      (E) 0.900

7. I flip the same coin three times. I win \$2 if I obtain exactly 1 tail or 3 tails. Otherwise, I lose \$1.50. The coin is biased: the probability of obtaining a tail is 0.4. If the random variable  $X$  denotes the amount won after the three flips, find  $\text{Var}(X)$ . (Note that the amount won is negative if I lose \$1.50.)

(A) 3.062      (B) 0.236      (C) 3.118      (D) 0      (E) 0.750

8. The random variable  $X$  has moment generating function

$$M_X(t) = \frac{1}{4}e^t + \frac{3}{4}e^{-t}$$

Find the mean of  $X$ .

(A)  $\frac{1}{4}$       (B) 1      (C)  $\frac{1}{2}$       (D)  $\frac{3}{4}$       (E)  $-\frac{1}{2}$

9. In a communication system, there is an average of one error in transmission every 10 seconds. If the errors follow a Poisson process, what is the probability that there will be more than one error in a transmission that lasts for 30 seconds?

(A)  $1 - 2e^{-1}$       (B)  $1 - e^{-1}$       (C)  $1 - 4e^{-3}$       (D)  $1 - 3e^{-3}$       (E)  $1 - e^{-3}$

10. Let  $X$  be a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{3} & 0 \leq x < 1 \\ \frac{1}{2} & 1 \leq x < 1.5 \\ 1 & x \geq 1.5 \end{cases}$$

Find the conditional probability that  $X$  is greater than 1.2 given that  $X$  is greater than 0.99.

- (A)  $\frac{1}{6}$       (B)  $\frac{1}{2}$       (C)  $\frac{3}{4}$       (D)  $\frac{3}{2}$       (E)  $\frac{2}{3}$

11. The probability mass function of a random variable  $X$  is given by

$x$	0	1	2	3
$f(x)$	$c$	$c/3$	$2c$	$5c/2$

where  $c$  is a constant. Find  $P(1.5 \leq X < 3)$ .

- (A)  $\frac{18}{35}$       (B)  $\frac{30}{35}$       (C)  $\frac{2}{5}$       (D)  $\frac{12}{35}$       (E)  $\frac{4}{5}$

12. Joe Blow did not study for this exam, and he is choosing his answers completely at random. What is the probability that he will get exactly 3 correct answers in the first 6 questions?

- (A)  $\binom{6}{3} 2^{-5}$       (B)  $\frac{4^4}{5^5}$       (C)  $\frac{1}{2^6}$       (D)  $\frac{1}{2}$       (E)  $\frac{5}{16}$