ADM 2304 Fall 2008

Midterm solutions

1.

Test and CI for One Proportion

```
Test of p = 0.045 vs p > 0.045
                                    95%
                                  Lower
Sample   X     N  Sample p        Bound  Z-Value  P-Value
1       90  1000  0.090000     0.075114     6.86    0.000
```

(a)

Ho: p = .045; Ha: p > .045

Z = (.09 – 045) / sqrt(.045*.955/1000) = .045 / .066 = 6.86

Reject Ho since Z > 1.645

Conclude Green Party has increased support.

The 2-proportions test is *incorrect*, since .045 is not a sample result but it is based on the entire population of voters) but half-marks if correctly done:

Test and CI for Two Proportions

```
Sample   X     N  Sample p
1       90  1000  0.090000
2       45  1000  0.045000

Difference = p (1) - p (2)
Estimate for difference:  0.045
95% lower bound for difference:  0.0266192
Test for difference = 0 (vs > 0):  Z = 4.01  P-Value = 0.000
```

(b)

P(Z>6.86) = .000

(c)

1-sided CI:

At least .09 – 1.645 * sqrt(.09*.91/1000) or .09 - 1.645 * .0091

At least .09 - .015 or .075 = [.075, 1.00]

Accept: At most .09 + .015 or [0, .105], even though this is more appropriate for the < Ha.

2-sided CI (half-marks)

.09 +/- 1.96 * .0091

.09 +/- .018

[.072, .108]

(d)

N = .09 * .91 * (1.96/.01)^2 = 3146

2. *Best to divide data by 1000 to get units of thousands of dollars.*
(a)
Two-Sample T-Test and CI

```
Sample   N   Mean  StDev  SE Mean
1       60   92.0   31.0      4.0
2       40  108.0   23.0      3.6

Difference = mu (1) - mu (2)
95% CI for difference:  (-26.7339, -5.2661)
T-Test of difference = 0 (vs not =): T-Value = -2.96  P-Value = 0.004  DF = 96
```

Two-Sample T-Test and CI

```
Sample   N   Mean  StDev  SE Mean
1       60   92.0   31.0      4.0
2       40  108.0   23.0      3.6

95% CI for difference:  (-27.3789, -4.6211)
T-Test of difference = 0 (vs not =): T-Value = -2.79  P-Value = 0.006  DF = 98
Both use Pooled StDev = 28.0906
```

Both samples > 30—there is no need to assume equal variances as we know the d.f. for t-distribution will be large and we can use the normal approximation.

-Ho: mu1-mu2=0, Ha: mu1-mu2≠ 0
-z = **-2.96** or -2.79, depending on assumption of unequal or equal variances
-reject Ho if |z| > 2.57 or 2.58
-reject Ho since z > 2.58, conclude average incomes are different

(b)
2 * P(z > 2.96) = 2 * .0015 = **.003**
2 * P(z > 2.79) = 2 * .0026 = .005

(c) Units are thousands of dollars:
+/- 16 +/- 1.96 * sqrt(31^2/60 + 23^2/40)
= 16 +/- 1.96 * 5.41 = 16 +/- 10.6 = **[5.4, 26.6]** *based on normal approx.*
Minitab uses t(.025) based on df=96 to obtain **(-26.7339, -5.2661)**

Or calculate pooled stdev of 28 and
+/- 1.96 * sqrt(28^2/60 + 28^2/40) = +/- 1.96* 5.72 = +/- 11.2
We have 16 +/- 11.2 = [4.8, 27.2]

(d)
Since both samples are large, we only need to assume populations are not extremely skewed. Or assume that sample means based on n=40 or 60 have *sampling distributions* that are *normally* distributed (this requires the first assumption above). Income data tend to be positively skewed and hence not normally distributed. The large sample sizes do not imply anything about the population distributions, only that the sample means tend to be normally distributed.

3.
(a)
The same 14 offices collected data before and after the program. The data must be paired.

(b)
Since data are matched pairs, we look only at the boxplot of the differences. Since this looks symmetric and there are no outliers, it is reasonable that these differences came from a normally distributed population. The best test is a parametric test--the paired t-test.
If answer was "independent samples" in (a), then we should look at the two boxplots (before and after). Given the outlier in the "before" boxplot, the best test is the non-parametric test (Mann-Whitney).

(c)
Ho: average difference is zero, Ha: average difference > zero
T=211.643/75.869= 2.79, where sterr is 283/sqrt(14).
Reject Ho if t| > 1.77 based on alpha=.05 and 13 d.f.
Decide to reject Ho at .05 level, conclude program was effective at reducing avg call
Only 1 mark for a 2-sample t-test.

(d)
Appropriate 1-sided CI is "at least 211 – 1.77*75" or at least 78. This excludes zero.
Inappropriate 1-sided CI is "at most 211 + 1.77*75" or at most 344. This includes zero.
2-sided CI would be 211 +/- 2.16*75 or 211 +/- 166 or [45 , 377]. This excludes zero.
(e)
Wilcoxon statistic is the sum of <u>ranks</u>, corresponding to the <u>positive</u> differences.

4.
(a)
Ho: variances are the same, Ha: variances are different (.5 marks)
-Alpha = .10 (since F(.05) tables given)
-F = (.0209/.0117)^2 = 3.19
-Reject Ho if F > 1.61 (based on 44 and 49 d.f. and alpha/2 = .05)
Conclude difference in volatility (.5 marks)

-if F calc as 1st sample var / 2nd sample var = (.0117/.0209)^2 = .31, then we need two critical values (upper F(.05) = 1.63 based on 49 and 44 d.f. and lower critical value F(.95) = 1/1.61 = .621).
-reject Ho if F > 1.63 or F < .621.

(b)
Daily changes in DJI should be normally distributed to justify a calculation of an F-statistic.