MIDTERM 1-MATH 1348-2008

NAME and I.D.# Solutions, Version 1

Instructions- The exam consists of 5 questions. There are a total of 25 possible points. The value of each question is as indicated. All work which you want graded must be clearly marked on the sheets provided. Make sure your name and student ID are *clearly written* above. **No notes, no scrap paper, no calculators, ABSOLUTELY NO CELL PHONES**

Question 1 (5 points)

Recall that, on the island of knights and knaves, all inhabitants are either knights, who always tell the truth, or knaves, who always lie. Suppose that you meet 2 inhabitants of this island, named A and B.

A says "I am a knave if and only if B is a knight"

B says "That's false".

What type of inhabitants are A and B? You must explain your answer.

Suppose A is a knave. Then his statement is false, and hence B is a knave.

Suppose A is a knight. Then his statement is true, and hence B is a knave.

Since, in either case, B is a knave, we conclude B is a knave.

Since B is a knave, his statement is false.

Hence what A says is true. So A is a knight.

Question 2 (3 points)

Consider the following truth table for the formula X, built from the variables p, q and r. Find a formula X corresponding to this truth table.

p	q	r	X
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

$(p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r)$

Question 3 (6 points) Part 1-Translate the following argument into propositional logic. Do not determine whether it is valid

If Ottawa did not win the Stanley cup, then Toronto was miserable only if Toronto came in last place. Toronto was miserable even though they did not come in last place. Therefore Ottawa won the Stanley cup.

- Use the following propositional letters in your translation:

  A: Ottawa won the Stanley Cup. B: Toronto was miserable.
  C: Toronto came in last place.

Part 2-Determine whether the following argument is valid. Explain your answer.

$$\begin{array}{c} B \Rightarrow (A \wedge \neg D) \\ \neg C \Rightarrow (B \Rightarrow D) \\ \underline{A \vee \neg B} \\ A \Rightarrow (B \vee C) \end{array}$$

Part 1 $\neg A \Rightarrow (B \Rightarrow C)$

$$\begin{array}{c} B \wedge \neg C \\ \hline A \end{array}$$

Part 2

A	B	C	D
T	F	F	T

This makes all premises true, and conclusion false.

Question 4 (6 points) Part 1- Let n be a positive integer. Prove that n is even if and only if $7n+4$ is even.

Part 2- Translate the following sentence into predicate logic, using predicates as indicated:

Only Giants fans are happy, but no Patriots fans are miserable.

Let $G(x)$=x is a Giants fan. Let $P(x)$=X is a patriots fan. Let $H(x)$=x is happy. Let $M(x)$=x is miserable.

Part 1- First show: If n is even, then $7n+4$ is even

Direct Proof: If n is even, then $n=2m$. So $7n+4=$
$7(2m)+4 = 14m+4 = 2(7m+2)$ which is even.

Now show: If $7n+4$ is even, then n is even.

Indirect proof: Instead show: If n is odd, then $n=2m+1$. So $7n+4 = 7(2m+1)+4 = 14m+11 = 2(7m+5)+1$ which is odd.

Part 2 $\forall x\,(H(x) \Rightarrow G(x)) \wedge \neg\exists(P(x) \wedge M(x))$

Question 5 (5 points) Prove that:

$$\underbrace{(A \cup B) \setminus (A \cap B)}_{X} = \underbrace{(A \setminus B) \cup (B \setminus A)}_{Y}$$

Suppose $x \in X$. So $x \in A$ or $x \in B$, but $x \notin A \cap B$.
There are 2 cases. ① Suppose $x \in A$. Since $x \notin A \cap B$,
we know $x \notin B$. So $x \in A \setminus B$. So $x \in Y$

② Suppose $x \in B$. Since $x \notin A \cap B$, we know $x \notin A$.
So $x \in B \setminus A$. So $x \in Y$.

Since one of these two cases must hold, we know
$x \in Y$. We have thus shown:

if $x \in X$, then $x \in Y$

We must also show:

if $y \in Y$, then $y \in X$.

Suppose $y \in Y$. Then $y \in A \setminus B$ or $y \in B \setminus A$.

2 cases:

① Suppose $y \in A \setminus B$. So $y \in A$ and $y \notin B$.
So $y \in A \cup B$ and $y \notin A \cap B$
So $y \in (A \cup B) \setminus (A \cap B) = X$.

② Suppose $y \in B \setminus A$. So $y \in B$ and $y \notin A$.
So $y \in A \cup B$, and $y \notin A \cap B$. So
$y \in (A \cup B) \setminus (A \cap B) = X$.

Since one or other case must hold, we know
$y \in X$. $\square$

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NAME and I.D.# Solutions, Version 2

INSTRUCTIONS- The exam consists of 5 questions. There are a total of 25 possible points. The value of each question is as indicated. All work which you want graded must be clearly marked on the sheets provided. Make sure your name and student ID are *clearly written* above. **No notes, no scrap paper, no calculators, ABSOLUTELY NO CELL PHONES**

Question 1 (5 points)

Recall that, on the island of knights and knaves, all inhabitants are either knights, who always tell the truth, or knaves, who always lie. Suppose that you meet 2 inhabitants of this island, named A and B.

A says "I am a knight if and only if B is a knave"

B says "That's true"

What type of inhabitants are A and B? You must explain your answer.

Suppose A is a knight. So his statement is true. Hence B is a knave.

Suppose A is a knave. So his statement is false. Hence B is a knave.

Since B is a knave in either case, B must be a knave.

Since B is a knave, what he says must be false. So A is lying. So A is a knave

Question 2 (3 points)

Consider the following truth table for the formula X, built from the variables p, q and r. Find a formula X corresponding to this truth table.

p	q	r	X
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

$(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$

Question 3 (6 points) Part 1-Translate the following argument into propositional logic. Do not determine whether it is valid

Toronto was miserable only if Ottawa won the Stanley cup or Toronto came in last place. Toronto came in last place, but Ottawa did not win the stanley cup. Therefore Toronto was not miserable.

- Use the following propositional letters in your translation:

  A: Ottawa won the Stanley Cup. B: Toronto was miserable.
  C: Toronto came in last place.

Part 2-Determine whether the following argument is valid. Explain your answer.

$$\begin{array}{c} A \wedge (\neg B \vee C) \\ \neg A \Rightarrow (B \Rightarrow C) \\ \underline{A \Rightarrow D} \\ (B \wedge C) \Rightarrow \neg D \end{array}$$

Part 1

$$\begin{array}{c} B \Rightarrow (A \vee C) \\ \underline{C \wedge \neg A} \\ B \end{array}$$

Part 2

A	B	C	D
T	T	T	T

This makes all premises true, and conclusion false. So argument is not valid

Question 4 (6 points)

Part 1-Let n be a positive integer. Prove that n is odd if and only if $5n + 6$ is odd.

Part 2- Translate the following sentence into predicate logic, using predicates as indicated:

If any Giants fans are happy, then only Patriots fans are miserable.

Let G(x)=x is a Giants fan. Let P(x)=x is a patriots fan. Let H(x)=x is happy. Let M(x)=x is miserable.

Part 1 - Suppose n is odd. Then $n=2m+1$.

So $5n+6 = 5(2m+1)+6 = 10m+11 = 2(5m+5)+1$

So $5n+6$ is odd.

To prove the other direction, we use an indirect proof. So we show:

if n is even, then $5n+6$ is even.

Suppose n is even. So $n=2m$. So $5n+6 = 10m+6 = 2(5m+3)$ which is even.

Part 2

$$\exists x(G(x) \wedge H(x)) \Rightarrow \forall x(M(x) \Rightarrow P(x))$$