3 August, 2011
Math 1107, Section A

TEST 4

1. Consider the matrix [10 marks]: $A = \begin{bmatrix} 1 & 3 & 6 & 2 & 4 \\ 0 & 1 & 2 & 1 & 1 \\ 2 & 2 & 4 & 0 & 4 \end{bmatrix}$

   (a) Find a basis for the column-space of A [6 marks].

   (b) What is the rank of A [2 marks]?

   (c) Does $basis\{ColA\}$ also form a basis for R^3? Explain [2 marks].

2. Let $T\colon R^4 \to R^3$ be the linear transformation defined by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{bmatrix} 2x_1 - 4x_3 \\ x_2 - x_3 + 3x_4 \\ x_1 + x_2 - 3x_3 + 3x_4 \end{bmatrix}.$$

   (a) Find the image of the vector, $\vec{x_D} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ by applying the given transform, T [2 marks].

   (b) Find the standard matrix of T [3 marks].

   (c) Find the image of $\vec{x_D}$ (above) by matrix multiplication of the standard matrix of T and x_D. Compare your answer to (a) [2 marks].

   (d) Is the vector $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ in the range of T? Explain [4 marks].

3. Consider the set of vectors, X, where $\vec{x_1} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \vec{x_2} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}, \vec{x_3} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$.

   (a) Confirm that this set is a basis for R^3, and explain your answer [2 marks].

   (b) Perform *Gram-Schmidt Orthogonalization* on the set X to create and orthogonal basis, $V = \{v_1, v_2, v_3\}$ [10 marks].

   (c) Check your answer (*i.e.* show that all the vectors in V are in fact orthogonal) [3 marks].