Exam #2 Fall 2006

→ Solution ←

1) 1) Given r, t_0, and t_1 all consumers choose c_0 and c_1 to maximize utility subject to their lifetime budget constraint.

2) Given r, G_0, and G_1, the government chooses T_0 and T_1 that satisfy its intertemporal budget constraint

3) r is such that the credit market clears [or is in equilibrium]

1) ⇒ $MRS_{c_0 c_1} = 1+r$

$$c_0 + \frac{c_1}{1+r} = y_0 - t_0 + \frac{y_1 - t_1}{1+r}$$

for both consumers [they are identical]

2) ⇒ T_0 and T_1 solve $G_0 + \frac{G_1}{1+r} = T_0 + \frac{T_1}{1+r}$

3) ⇒ $S_0^P + S^G = 0$

2) $MRS_{c_0 c_1} < 1+r$ cannot be an equ. because, in this case, the value attached by consumers to c_0 in terms of c_1 is less than the "market" value $\frac{p_0}{p_1} = 1+r$. Should $\downarrow c_0$ (and $\uparrow c_1$) [sell c_0] until $MRS_{c_0 c_1} = 1+r$

3) government's BC with $G_0 = 5$
$G_1 = 12$
$B_0 = 1$

note: 1st $G_0 = T_0 + B_0$ (in period 0)

$$\therefore \quad T_0 = G_0 - B_0$$
$$= 5 - 1$$
$$\boxed{T_0 = 4}$$

2nd $G_1 + (1+r)B_0 = T_1$ (in period 1)

$$12 + (1+r)1 = T_1$$

$$\therefore \quad \boxed{T_1 = 13 + r}$$

since tax burden is shared equally:

$$t_0 = \frac{1}{2} T_0 \qquad t_1 = \frac{1}{2} T_1$$

$$\boxed{t_0 = 2} \qquad \boxed{t_1 = \frac{13 + r}{2}}$$

4) Ricardian Eqv. Theorem:

the timing of taxes does not matter as long as G_0 and G_1 are kept constant [under conditions that are met here]

$\therefore$ since $T_0 \neq 2 \times t_0 = 5 = G_0$

$T_1 = 2 \times t_1 = 12 = G_1$

T_1 and T_0 satisfy the government's B.C.

$\Rightarrow$ we will have same C_0^* and r^*, etc...

5) consumers: [condition 1) in Question 1)]

$$MRS_{c_0 c_1} = 1+r \qquad c_0 + \frac{c_1}{1+r} = y_0 - t_0 + \frac{y_1 - t_1}{1+r}$$

$$\frac{\frac{\partial u}{\partial c_0}}{\frac{\partial u}{\partial c_1}} = \frac{\frac{1}{c_0}}{0.8\frac{1}{c_1}} = 1+r$$

$$\frac{c_1}{0.8\, c_0} = 1+r$$

$$\boxed{\frac{c_1}{1+r} = 0.8\, c_0} \quad (*)$$

also: $c_0 + \frac{c_1}{1+r} = 12.5 - 2.5 + \frac{16-6}{1+r}$

from question 4) → [condition 2) in Question 1)]

$$\boxed{c_0 + \frac{c_1}{1+r} = 10 + \frac{10}{1+r}}$$

with $(*)$ $\quad c_0 + 0.8\, c_0 = 11.125 + \frac{10}{1+r}$

$$c_0^* = \frac{1}{1.8}\left[10\left(\frac{2+r}{1+r}\right)\right]$$

3) in question 1) $\Rightarrow$ $S_0^P + S_0^G = 0$

note $S_0^G = -B_0 = 0$ if used $t_0 = 2.5$, $t_1 = 6$

need to show that $S_0^P = 0$

could do it with $B_0 = 1$ but different taxes

$t_0 = 2$
$t_1 = 6.6$

$$S_0^P = \underbrace{y_0 - t_0 - c_0 + y_0 - t_0 - c_0}_{\text{identical decisions}}$$

$$= 2(y_0 - t_0 - c_0)$$

$$= 2\left[10 - \frac{1}{1.8}\left[10\,\frac{(2+r)}{1+r}\right]\right]$$

with $r = 0.25$

$$= 20 - \frac{2}{1.8}\cdot\left[10\,\frac{2.25}{1.25}\right]$$

$$= 20 - 20 = 0$$

$$= 0.$$

$\therefore S_0^P = 0$ as we needed to show

note: actual $S_0^P = 1$ with $t_0 = 2$

6) Consumers are identical → same preferences, same income

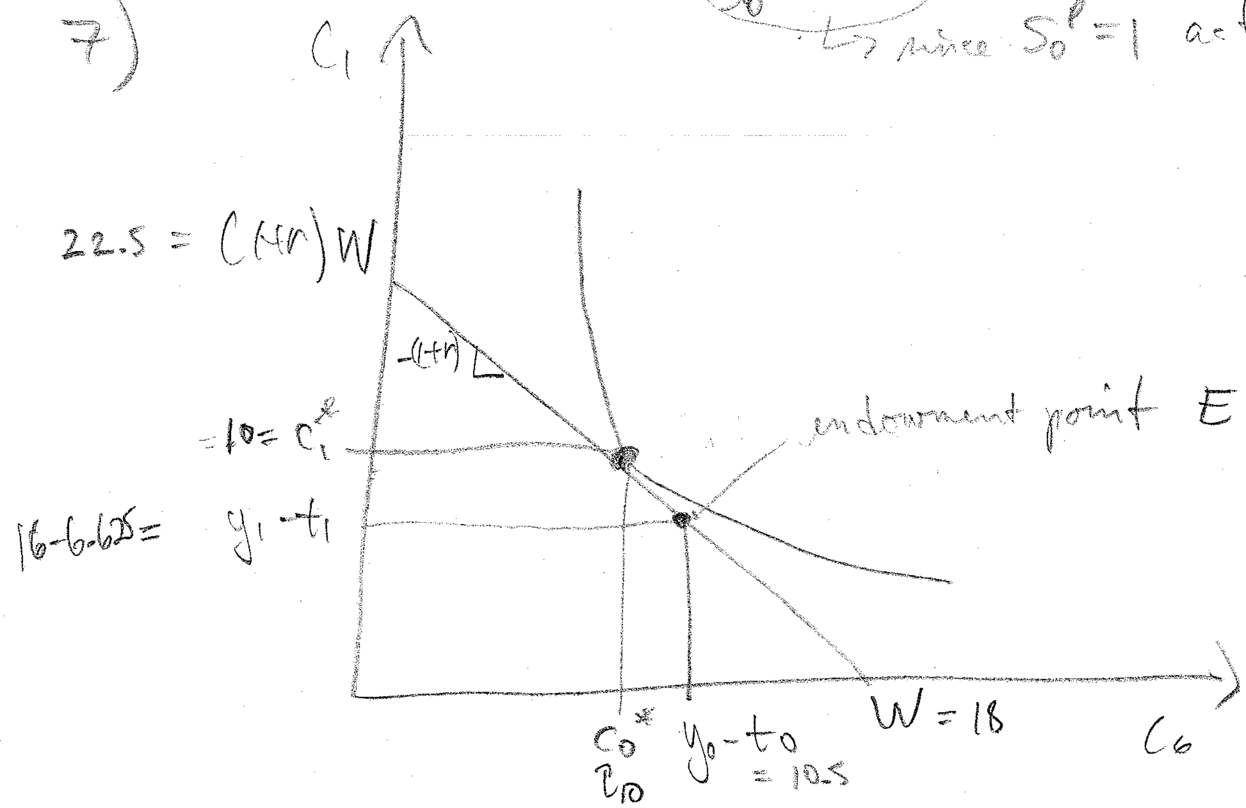
↳ . they take the same decisions

. no reasons to trade

$S_0 = 0.5$

$\hookrightarrow$ since $S_0^P = 1$ actually

7)



8) $Y_0 = 2 \cdot y_0 \quad \therefore \quad y_0 = \frac{1}{2} Y_0$

$$W = y_0 + \frac{y_1}{1+r} - \left[t_0 + \frac{t_1}{1+r} \right]$$

$$= \frac{1}{2} Y_0 + \frac{y_1}{1+r} - \frac{1}{2}\left[G_0 + \frac{G_1}{1+r} \right]$$

$$W = \frac{1}{2} Y_0 + \frac{16}{1+r} - 2.5 - \frac{6}{1+r}$$

$$W = \frac{1}{2} Y_0 + \frac{10}{1+r} - 2.5$$

from 5) $\quad c_0^* = \frac{1}{1.8} W$

$$c_0^* = \frac{1}{2 \times 1.8} Y_0 + \frac{10/1.8}{1+r} - 2.5/1.8$$

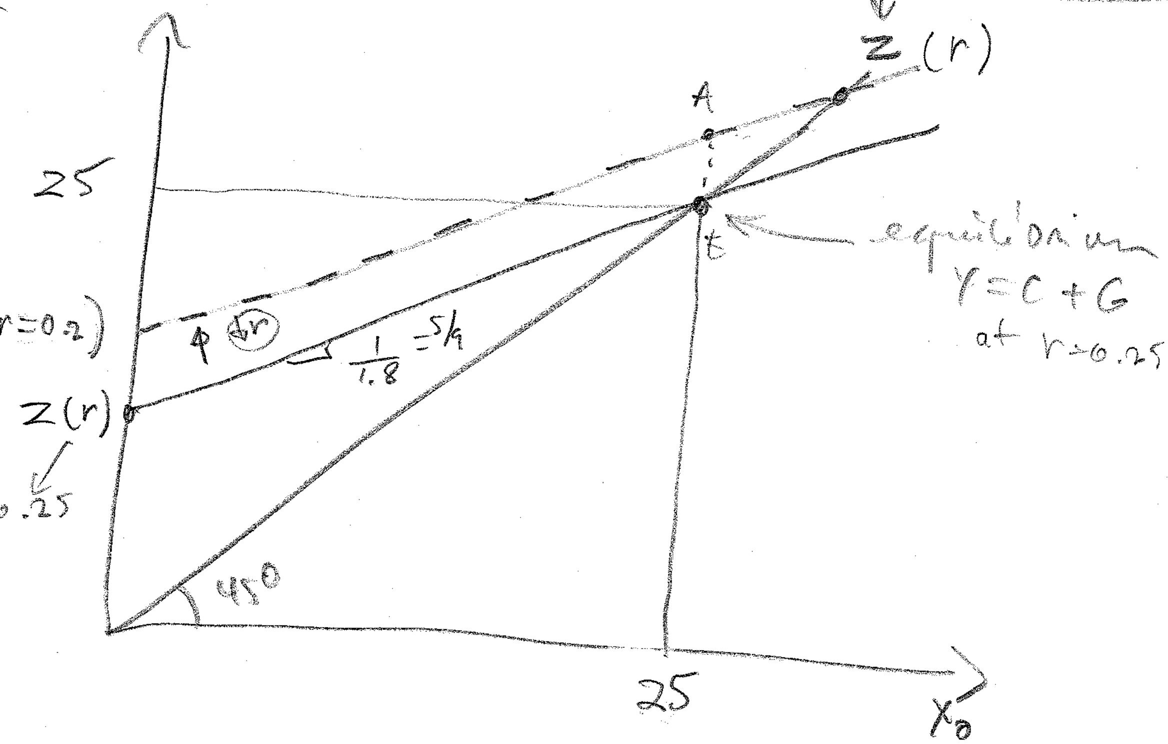
$C_0 = 2 \cdot c_0$

$$\boxed{C_0 = \frac{1}{1.8} Y_0 + \frac{20/1.8}{1+r} - 5/1.8}$$

$AD = C_0 + G_0$

$$\boxed{AD = \frac{1}{1.8} Y_0 + \underbrace{\frac{20/1.8}{1+r} - \frac{5}{1.8} + 5}_{Z(r)}}$$

why this is required on answer



the Equilibrium is at point E where $Y = C + G$

when $r = 0.2$, $C_0 + G_0$ is at point A $> Y_0$

so $C_0 + G_0 > Y_0$ ie the goods market is not in equilibrium (r has to ↑ to reduce C_0 until $Y_0 = C_0 + G_0$)

9)

Correlation is not an indication of causality

for example $\downarrow G_0 \Rightarrow \downarrow r$ and $\uparrow C_0$

but $\downarrow r$ does not cause $\uparrow C_0$, it is the $\downarrow G_0$ that does cause $\uparrow C_0$

10) This is equivalent to a $\downarrow$ in β

ie! $u = \ln(C_0) + \underline{0.8} \ln(C_1)$

$$\downarrow$$

$u = \ln(C_0) + \underline{0.5} \ln(C_1)$ say

$MRS_{C_0 C_1} = 1+r$

sick: $\dfrac{C_1^S}{0.5\, C_0^S} = 1+r$

not sick: $\dfrac{C_1^{NS}}{0.8\, C_0^{NS}} < \dfrac{C_1^S}{0.5\, C_0^S}$

C_1^{NS} must $\uparrow$ and C_1^S must $\downarrow$
to restore equilibrium ($C_0^{NS} \downarrow$ and $C_0^S \uparrow$)

$\therefore \quad C_1^{NS} > C_0^{NS}$ [$C_1^{NS} = C_0^{NS}$ with $r = 0.25$]

$\Rightarrow \quad 1 + r_{NEW} = \dfrac{C_1^{NS}}{0.8\, C_0^{NS}} > \dfrac{1}{0.8} = 1.25$

or: The sick consumer wants to $\uparrow C_0$ because he will care less about future consumption [because he is not sure to be alive then] Therefore, he needs to borrow from the other consumer. To convince the healthy consumer to increase saving, an $\uparrow$ in r is required