## labour supply in a two-period economy

→ We know that to get income or production to change endogenously, we need labour ($N_0$) to vary since $Y_0 = z_0 K_0^{\alpha} N_0^{1-\alpha}$ and $K_0$ is predetermined [ie $K_0$ is determined in periods before time 0]
current period

→ we already have determined labour demand:

$$w_0^d = MPN_0 = (1-\alpha) z_0 K_0^{\alpha} (N_0^d)^{-\alpha}$$

(the inverse labour demand actually)

∴ we need to look at labour supply to close the model.

→ labour supply:

↳ we already looked at labour supply in a static (one period) setting

ie: labour supply was determined by solving

$$\max_{c,l} u(c, l)$$
$$\text{s.t. } c + wl = wh + \pi - t$$

in a dynamic setting (2 periods), the problem is similar, except that the consumer can save labour income.

ie: the budget constraints that the consumer faces is:

$$c_0 + w_0 l_0 + s_0 = w_0 h + \pi_0 - t_0$$

$$c_1 + w_1 l_1 = w_1 h + \pi_1 - t_1 + (1+r) s_0$$

→ we can rewrite these two budget constraints into one intertemporal budget constraint (as we did before)

ie: $s_0 = w_0 h + \pi_0 - t_0 - w_0 l_0 - c_0$

$$\therefore \quad c_1 + w_1 l_1 = w_1 h + \pi_1 - t_1 + (1+r)\left[w_0 h + \pi_0 - t_0 - w_0 l_0 - c_0\right]$$

$$c_1 + (1+r) c_0 = w_1 h - w_1 l_1 + \pi_1 - t_1 + (1+r)\left[w_0 h + \pi_0 - w_0 l_0\right]$$

$$\boxed{c_0 + \frac{1}{1+r} c_1 = w_0 (h - l_0) + \pi_0 - t_0 + \frac{w_1 (h - l_1) + \pi_1 - t_1}{1+r}}$$

Therefore, the problem faced by the consumer is to maximize $u(c_0, l_0) + \beta u(c_1, l_1)$ subject to this intertemporal budget constraint.

solution: $MRS_{c_0 c_1} = 1+r$

cost and benefit of $\Delta l_0$ happen at time 0. Therefore, no intertemporal linkage. →

$$\underbrace{MRS_{l_0 c_0}}_{\text{value of leisure}} = \underbrace{w_0}_{\text{opportunity cost of leisure}}$$

$$MRS_{l_1 c_1} = w_1$$

ex: Suppose there are M identical consumers with utility $u = \ln(c_0) + \ln(l_0) + \beta[\ln(c_1) + \ln(l_1)]$

$$\therefore \quad MRS_{c_0 c_1} = \frac{c_1}{\beta c_0}$$

$$MRS_{l_0 c_0} = \frac{c_0}{l_0} \qquad MRS_{l_1 c_1} = \frac{c_1}{l_1}$$

$$c_0 + \frac{1}{1+r} c_1 = w_0 (\underbrace{h - l_0}_{n_0}) + \pi_0 - t_0 + \frac{w_1(\underbrace{h - l_1}_{n_1}) + \pi_1 - t_1}{1+r}$$

$\therefore$ we have:

$$\frac{c_0}{l_0} = w_0$$

$$\frac{M \cdot c_0}{M \cdot l_0} = w_0 \Rightarrow \frac{\overbrace{C_0}^{\text{aggregate consumption}}}{\underbrace{L_0}_{\text{aggregate leisure}}} = w_0$$

$$\therefore \boxed{\frac{C_0}{\underbrace{H}_{M \cdot h} - \underbrace{N_0^s}_{\text{total labour supply}}} = w_0}$$

(ie: $L_0 = H - N_0^s$)

also, similarly, $\frac{c_1}{\beta c_0} = 1+r$

$$\Rightarrow \frac{M c_1}{M \beta c_0} = 1+r \quad \therefore \boxed{\frac{C_1}{\beta C_0} = 1+r}$$

$$M c_0 + \frac{1}{1+r} M c_1 = \underbrace{M w_0 n_0 + M \pi_0}_{\to Y_0 \text{ (using income approach)}} - \underbrace{M t_0}_{\to T_0} + \frac{M w_1 n_1 + M \pi_1 - M t_1}{1+r}$$

$$\therefore \boxed{C_0 + \frac{1}{1+r} C_1 = Y_0 - T_0 + \frac{Y_1 - T_1}{1+r}}$$

note: the trade-off between $C_0$ and $C_1$ needs to satisfy the same conditions as before

$$\therefore \quad C_0 = \underbrace{C_0^*(r)}$$

↳ the same $C_0^*(r)$ that we have calculated before

Therefore, given $C_0^*(r)$, labour supply is simply given by

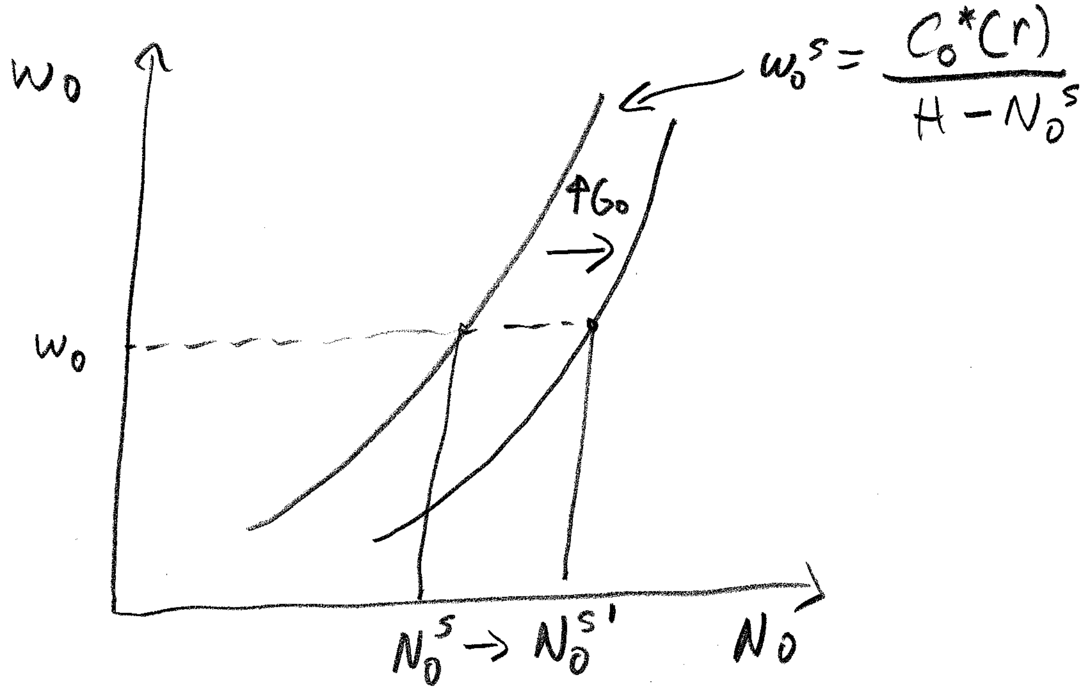
$$\boxed{w_0^s = \frac{C_0^*(r)}{H - N_0^s}}$$

→ note: $\uparrow w_0 \longrightarrow \uparrow N_0^s$

note: "events" in the economy affect labour supply through consumption decisions. Anything that changes $C_0^*$, changes $N_0^s$.

ex: $\uparrow G_0 \rightarrow \downarrow C_0$ ( because $\uparrow G_0 \rightarrow \underbrace{\uparrow T_0 \text{ or } \uparrow T_1}_{\downarrow W}$ ]

$\therefore$ given $w_0$, $N_0^s \uparrow$ (true for any $w_0$)

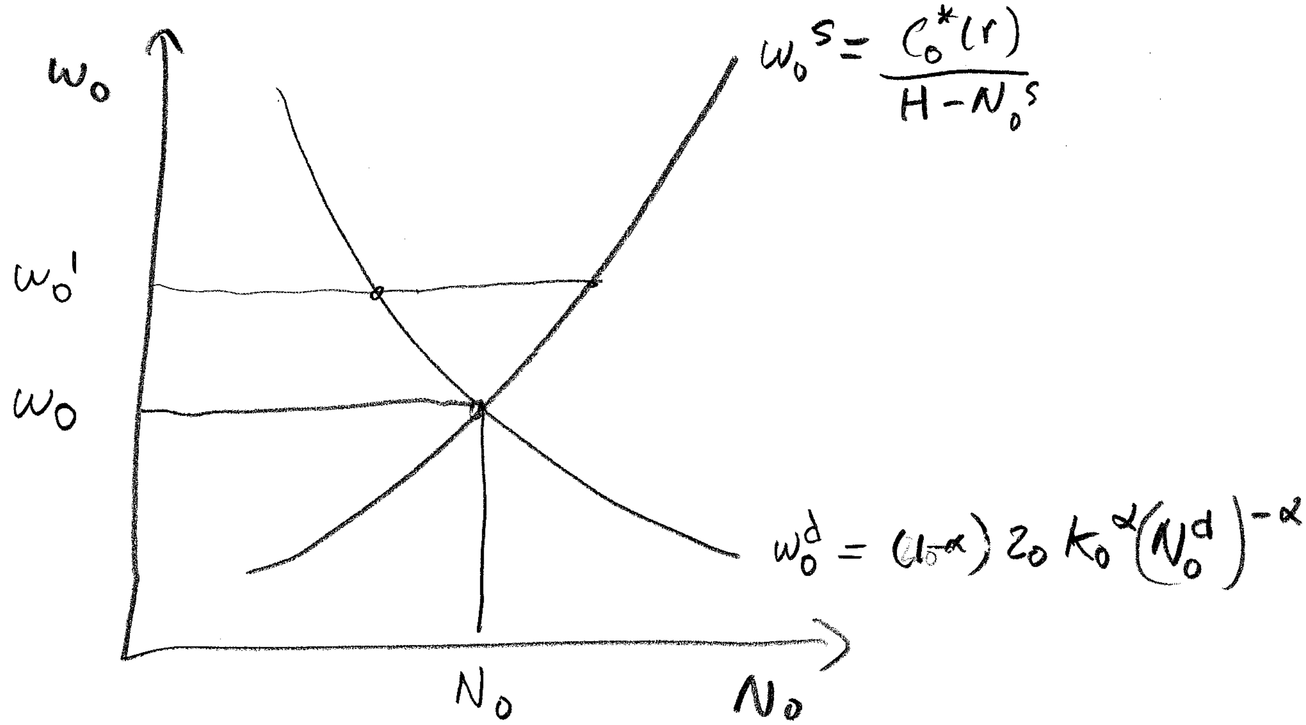



Similarly, given $w_0$,

$\uparrow r \rightarrow \uparrow \frac{p_{C0}}{p_{C1}}$ $\left(\frac{p_{C0}}{p_{C1}} = 1+r\right) \rightarrow \downarrow C_0 \rightarrow \uparrow N_0^s$

$\uparrow Y_1^e \rightarrow \uparrow C_0$ (consumption smoothing) $\rightarrow \downarrow N_0^s$
[ie $\uparrow Y_1 \rightarrow \uparrow C_1$]

with $w_0^s$ in hand, it is possible to determine the movements in $N_0 = N_0^s = N_0^d$ in equilibrium using the graph:

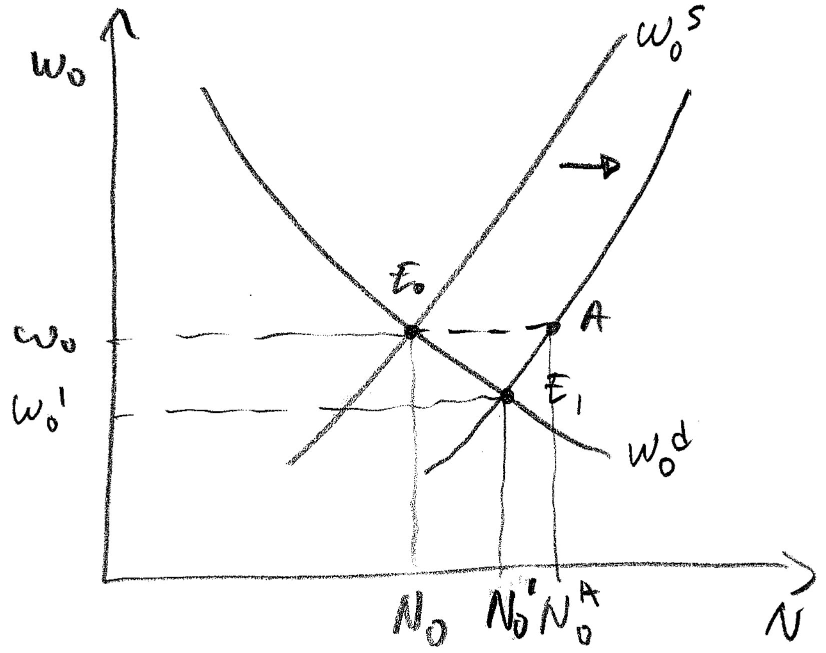



note: at $w_0'$, not an equilibrium: $N_0^s > N_0^d$
$\therefore$ excess supply.
($w_0'$ must fall) until $N_0^d = N_0^s$

→ We can use this graph to infer the relationship between aggregate supply $Y_0^s = z_0 K_0^\alpha N_0^{1-\alpha}$ and the interest rate "r"

ie: suppose the labour market is in equilibrium at $r_0$ so that $Y_0^s(r_0) = z_0 K_0^\alpha N_0^{1-\alpha}$. Now, suppose that $r_0 \uparrow$ to $r_0'$ for some reason, what happens to $Y_0^s(r_0')$?

ii:




$\uparrow r \longrightarrow \downarrow C_0^* \longrightarrow \uparrow N_0^s$ (given $w_0$)

(from $r_0$ to $r_0'$) (movement from $E_0$ to $A$)

at $A$, $N_0^d = N_0$ but $N_0^s \equiv N_0^A$

$\therefore$ excess supply ($N_0^A > N_0$)

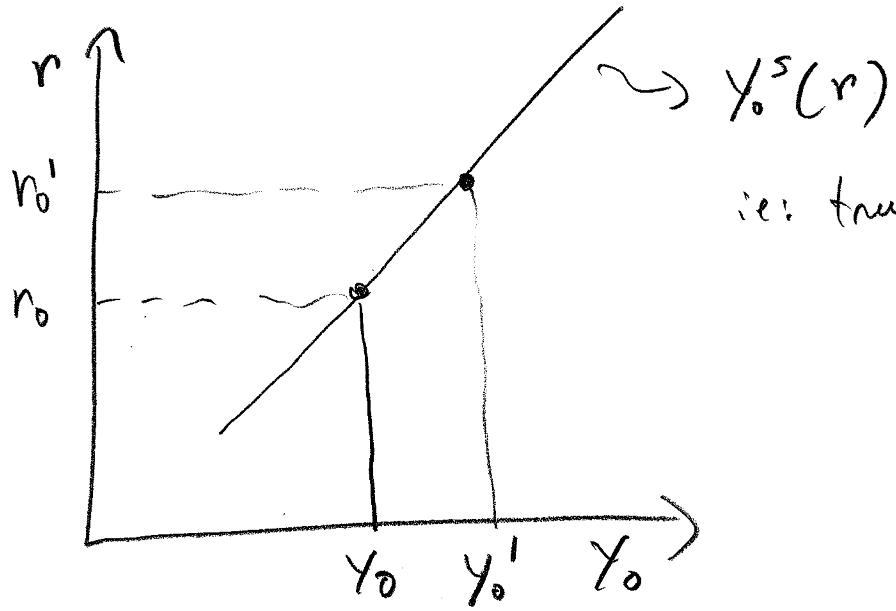
$\therefore$ $w_0$ must $\downarrow$ until $w_0'$

So the new $N_0$ is $N_0'$

$\therefore$ $Y_0^s(r_0') > Y_0^s(r_0)$

$= z_0 K_0^\alpha (N_0')^{1-\alpha} > z_0 K_0^\alpha (N_0)^{1-\alpha}$

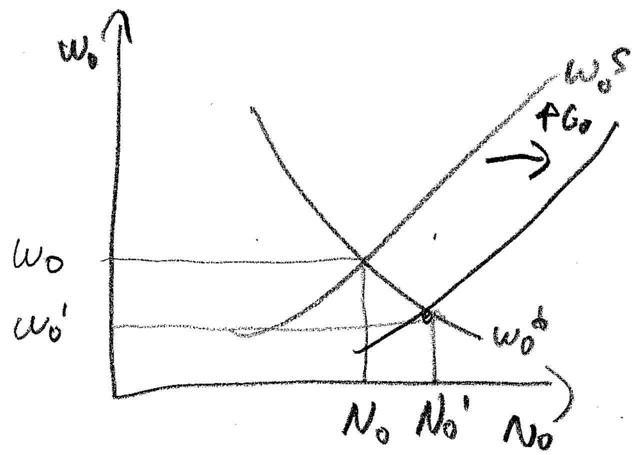
graphically, we have:




ie: true for any $\Delta r$

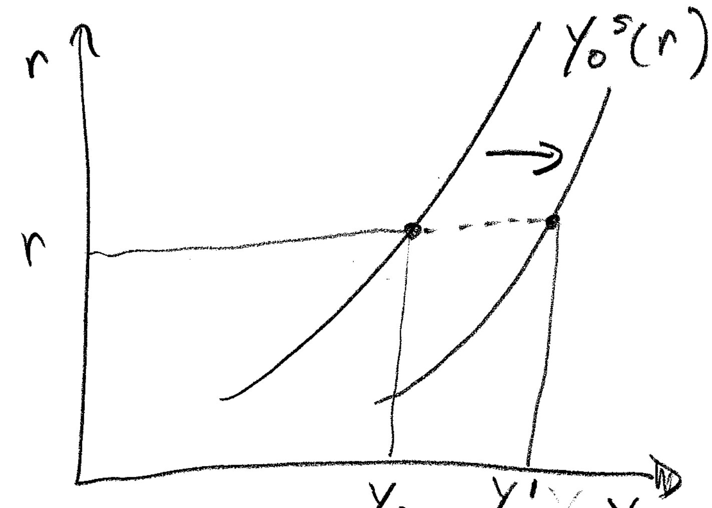
→ again, let's look at $\uparrow G_0$ to determine its impact on $Y_0^s(r)$

recall:

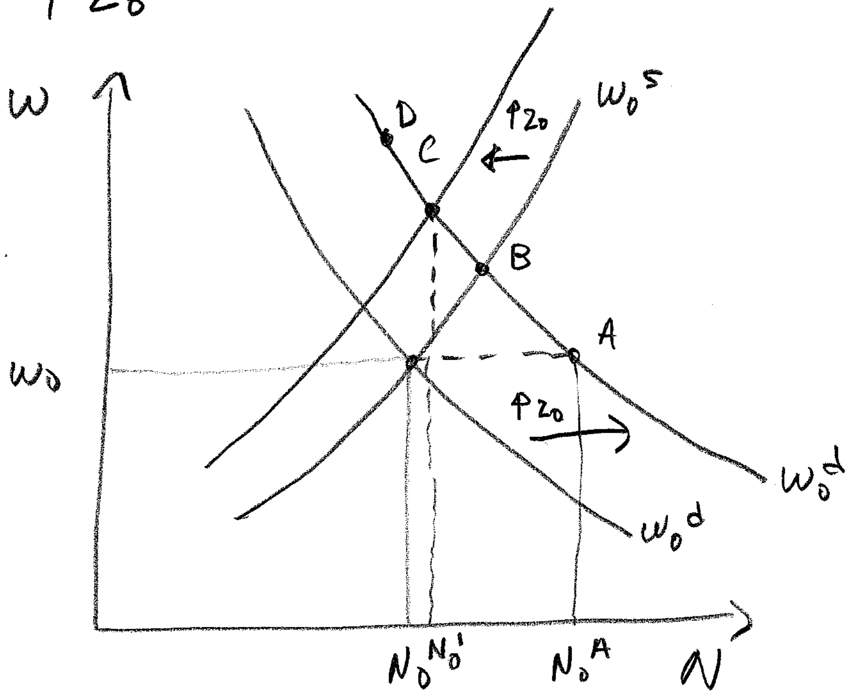



$\therefore$ given r, $\uparrow G_0 \rightarrow \uparrow N_0$ (because $C_0^* \downarrow$)

Thus, we have,



ex: $\uparrow z_0$




1) $\uparrow z_0 \rightarrow \uparrow MPN_0 \therefore N_0^d \uparrow$ given $w_0$

(point A) $\rightarrow N_0^d = N_0^A \therefore$ excess demand for labour

$\therefore w_0$ should $\uparrow$

ie: movement towards point B.

2) $\uparrow z_0 \rightarrow \uparrow Y_0$ ($\uparrow \pi_0$ given $w_0$)

$\therefore \uparrow C_0$ ($\uparrow C_1$) but $\uparrow C_0 \rightarrow \downarrow N_0$ given $w_0$ (even bigger excess demand)

equilibrium: $\boxed{\text{point C}}$

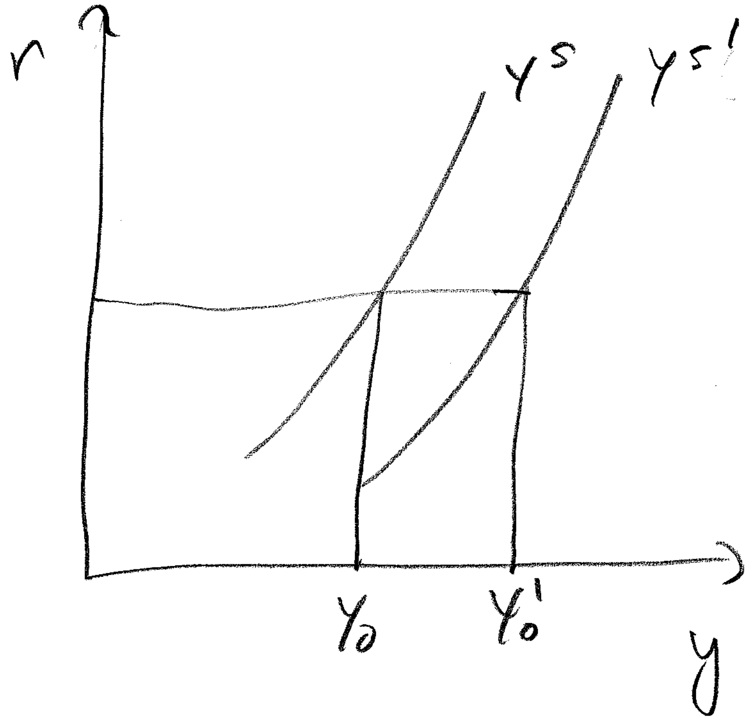
note: point D could also be an equilibrium depending on how strong is the backward movement in the inverse labour supply curve.

note: 1) = substitution effect

2) = income effect

if $N_0' > N_0$, substitution effect > income effect

$\therefore \uparrow Y_0$ when $z_0 \uparrow$ (given $r$)




(note: even if $N_0 \downarrow$, $Y_0$ still $\uparrow$ because $z_0 \uparrow$ and $\Delta z_0$ has a bigger impact on $Y_0$ than $\Delta N_0$)