Econ 303 | 12

Firms decisions in a two-period economy

→ Here, we introduce the notion of investment. Recall that the representative firm's production function depends on capital and labour.

↳ labour can be adjusted freely in any period. Capital must be installed in advance [think of building a factory]. Investment thus have an intertemporal dimension, just like saving does.

→ when the firm determines investment, it attempts to maximize the present value of profits [ie profits evaluated at period 0 prices]. Recall that the price of period 1 goods in terms of period-0 goods is $\frac{1}{1+r}$. Thus, the representative firm maximizes

$$V = \pi_0 + \frac{1}{1+r}\pi_1$$

profits at time 0 — "intertemporal" price — profits at time 1

↳ V is often referred to as the firm's value [this should be the firm's value on the stock market [fundamentals]]

→ given K_0,

$$\Pi_0 = \underbrace{z_0 F(K_0, N_0)}_{\text{output}} - \underbrace{w_0 N_0}_{\text{cost of labour}} - \underbrace{I_0}_{\text{gross investment}}$$

$$\Pi_1 = z_1 F(K_1, N_1) - w_1 N_1 - I_1$$

→ $K_1 = (1-d) K_0 + I_0$ → spending on capital units

assumption: $100 \times d$ % of the capital stock "depreciates" (become useless) after it has been used to produce goods (wear and tear)

∴ at the end of the year, only $(1-d)K_0$ is left of the K_0 units installed

∴ before K_1 can ↑ above K_0, the units that are broken (dK_0) must be replaced

↳ $I_0 = \underbrace{K_1 - K_0}_{\text{net investment}} + dK_0$ → spending to maintain the capital stock

gross investment → actual spending on capital goods

net investment → actual change in the capital stock

Note:

1) $\uparrow I_0 \longrightarrow \uparrow K_1$: this process as a cost or a benefit

↳ determine I_0 given K_1^*

2) $\uparrow I_1 \longrightarrow \uparrow K_2$

↳ but there is no period 2

$\therefore K_2^* = 0$

$K_2^* > 0$ not optimal

↳ costly to $\uparrow I_1$ but no benefit

$\therefore \quad I_1 = K_2 - K_1 + dK_1$

$I_1 = K_2^* - K_1 + dK_1$

$$\boxed{I_1 = \underbrace{-(1-d)}_{\ominus} K_1}$$

$\ominus \rightarrow$ this is divestment

in period 1, firms liquidate their capital stock. (ie sell $(1-d)K_1$)

↳ consumed

(3)

using 1) and 2), we have

$$\Pi_0 = z_0 F(K_0, N_0) - w_0 N_0 \overbrace{- K_1 + (1-d) K_0}^{-I_0}$$

$$\pi_1 = z_1 F(K_1, N_1) - w_1 N_1 + \overbrace{(1-d) K_1}^{-I_1}$$

$\therefore$

$$V = \overbrace{\left[z_0 F(K_0, N_0) - w_0 N_0 - K_1 + (1-d) K_0 \right]}^{\pi_0}$$

$$+ \frac{1}{1+r} \underbrace{\left[z_1 F(K_1, N_1) - w_1 N_1 + (1-d) K_1 \right]}_{\pi_1}$$

The firm's problem is to maximize V by choosing N_0, N_1, and K_1 **given r and w**

→ note: ΔN_0 only changes Π_0, ΔN_1 only changes π_1 → no intertemporal aspect in determining labour demand

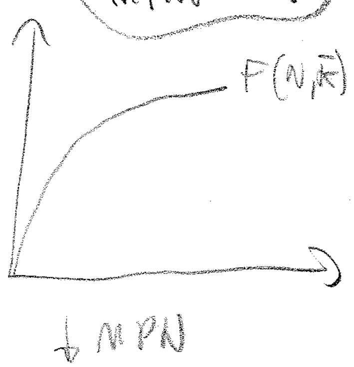
$\frac{\partial V}{\partial N_0}$

$\therefore$ the conditions determining labour demand are the same as the one we saw before

ie: $MPN_0 = w_0$

$MPN_1 = w_1$

$\uparrow N_0$ by 1
↳ $\uparrow V$ by MPN_0
$\downarrow V$ by w_0
$MPN_0 - w_0$



eg: MPN = benefit of $\uparrow N$

w = cost of $\uparrow N$

$MPN > w$ → Benefit > cost ⇒ $\uparrow N$ (which $\downarrow MPN$) until $MPN = w$

$MPN < w$ → benefit < cost → $\downarrow N$ → $\uparrow MPN$ until

→ Determining K_1 works in a similar manner except that the cost is incurred in period 0 while the benefit is in period 1:

Cost : $\uparrow K_1$ by 1\$ reduces Π_0 by 1\$
ie $\uparrow I_0$

Benefit : $\uparrow K_1$ by 1\$ increases Π_1 in two ways

1) increases output → that increase is measured by MPK_1

2) increases the capital stock sold in period 1 } → ie: $\uparrow \Pi_1$ by $(1-d)$

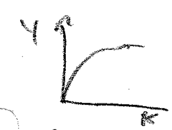
↳ another more general way to see this is that it reduces investment spending in period 1 [if there is a period 2]

∴ optimal value of K_1 is such that

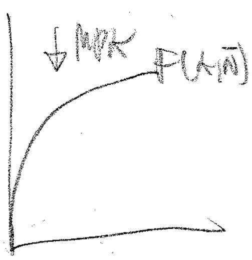
$$\frac{\partial \Pi_1}{\partial K_1} \rightarrow \underbrace{\overbrace{\frac{MPK_1 + 1 - d}{\underbrace{1+r}}}^{\text{Benefit}}}_{} = \overbrace{1}^{\text{cost}} \leftarrow \frac{\partial \Pi_0}{\partial K_1}$$

$\frac{P_{c1}}{P_{c0}} = \frac{1}{1+r}$

↳ present value of benefit (recall this happens at time 1, not time 0)



$\uparrow K \rightarrow \downarrow MPK$



again, if $\frac{MPK_1 + 1 - d}{1+r} > 1 \rightarrow \uparrow K_1 \rightarrow \downarrow MPK_1$ (until Benefit = cost)

if $\frac{MPK_1 + 1 - d}{1+r} < 1 \rightarrow \downarrow K_1 \rightarrow \uparrow MPK_1$ (until Benefit = cost) it follows that

$$MPK_1 + 1 - d = 1 + r$$

$$\underbrace{MPK_1}_{\substack{\text{benefit of}\\ \text{using an extra}\\ \text{unit of}\\ \text{capital}\\ \text{(ie } \Delta \text{ in output)}}} = \underbrace{r + d}_{\substack{\text{user cost}\\ \text{of capital}}}$$

user cost of capital ↳ suppose you borrow one unit of capital

- need to pay interest rate
- need to restore it to its initial state (pay the depreciate) before returning it to its owner

↳ cost paid to <u>use</u> one extra unit of capital

$\boxed{\text{benefit} = \text{cost}}$

⇒ this determines the demand for Investment in period 0

ex: say $Y_1 = z_1 K_1^{\alpha} N_1^{1-\alpha}$

Then $MPK_1 = \dfrac{\partial Y_1}{\partial K_1} = \alpha z_1 K_1^{\alpha-1} N_1^{1-\alpha}$

$$= \frac{\alpha z_1 K_1^{\alpha} N_1^{1-\alpha}}{K_1}$$

$$\boxed{MPK_1 = \alpha \frac{Y_1}{K_1}}$$

recall that $K_1 = (1-d)K_0 + I_0$

$$\therefore \quad MPK_1 = \frac{\alpha Y_1^e}{I_0 + (1-d)K_0}$$ ← expected output in period 1

as $MPK_1 = r + d$

$$\frac{\alpha Y_1^e}{I_0 + (1-d)K_0} = r + d$$

$$\Rightarrow \quad I_0 + (1-d)K_0 = \frac{\alpha Y_1^e}{r+d}$$

$$\boxed{I_0 = \underbrace{\frac{\alpha Y_1^e}{r+d} - (1-d)K_0}_{I(r,\, Y_1^e,\, K_0)}}$$

$$\quad\ominus\quad \oplus\quad \ominus$$

demand for goods from investment

not right actually but approximately right to make the point across

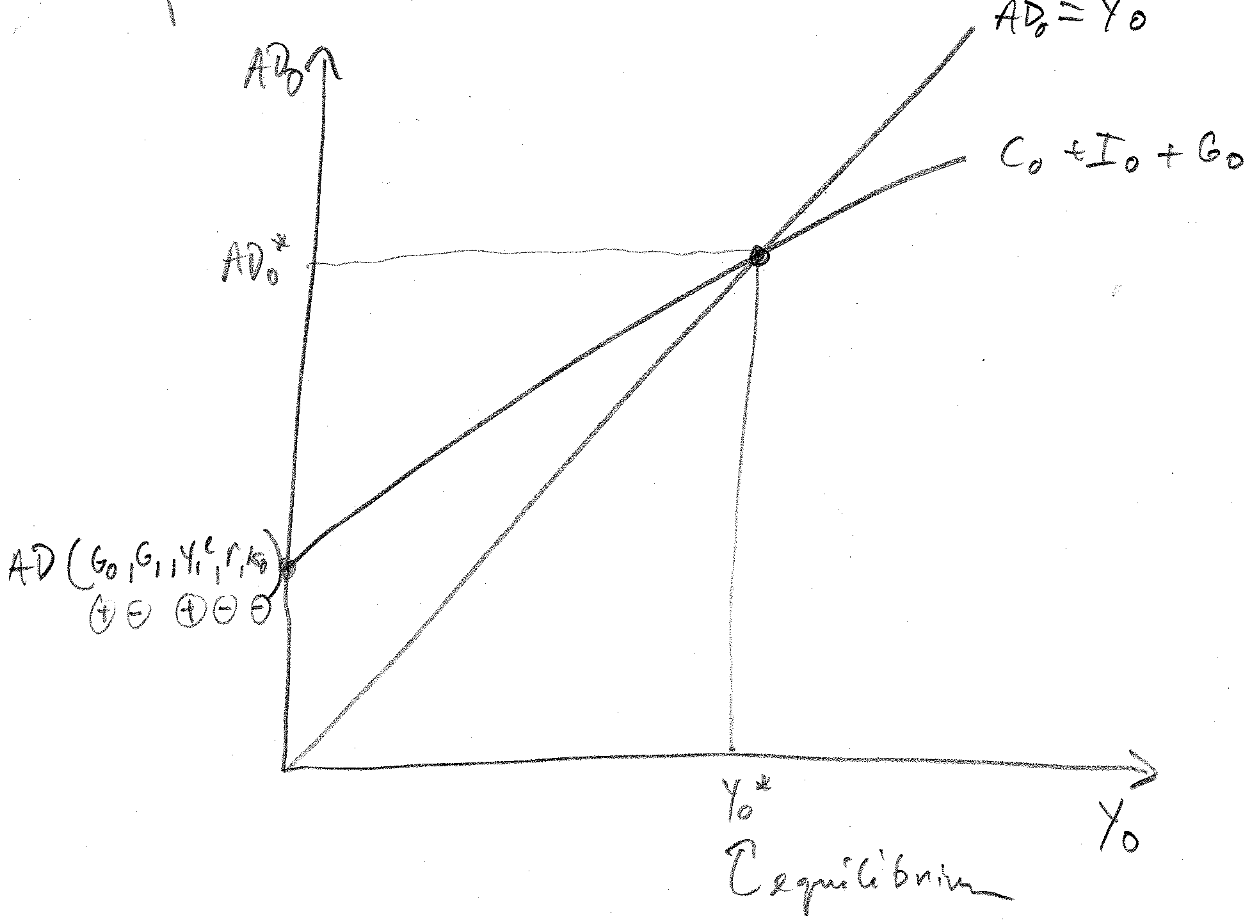
[assume ΔK_1 does not Δ Y_1 much]

Adding I_0 to $C_0 + G_0$ [see (10a) p.2]
we can get aggregate demand

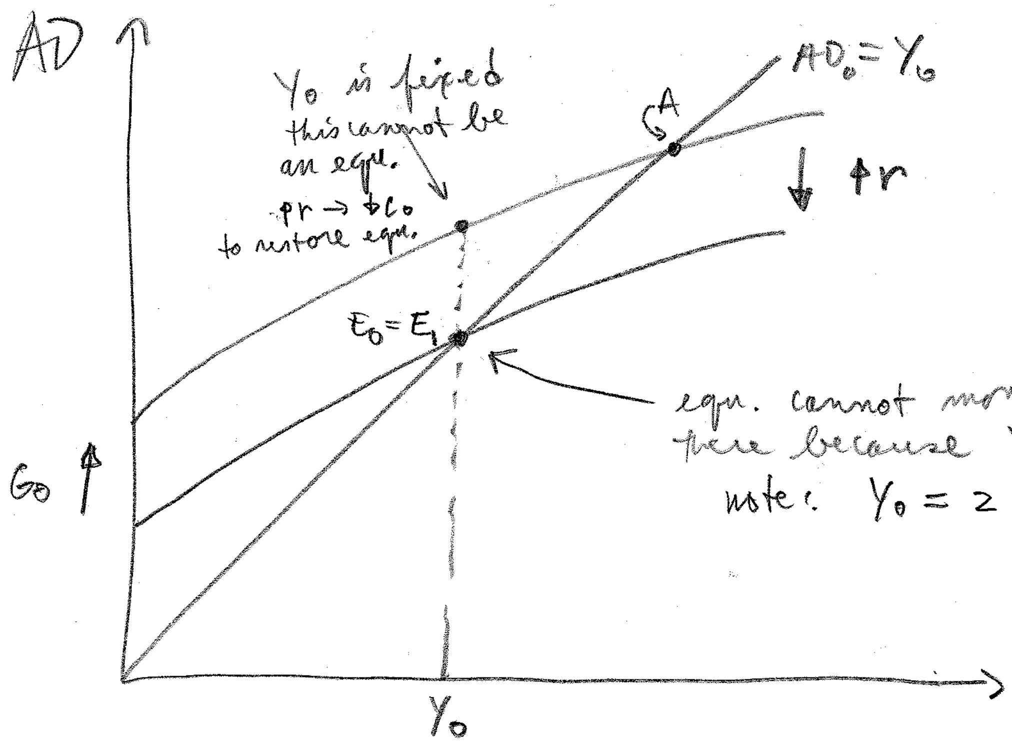
$$\underbrace{C_0 + I_0 + G_0}_{AD_0} = \frac{1}{1+\beta} Y_0 + \underbrace{\frac{\beta}{1+\beta} G_0 + \frac{1}{1+\beta}\frac{Y_1^e - G_1}{1+r} + \frac{\alpha Y_1^e}{r+d} - (1-d)K_0}_{AD(G_0,\, G_1,\, Y_1^e,\, r,\, K_0)}$$

$$\oplus \quad \ominus \quad \oplus \quad \ominus \quad \ominus$$

graphically,



now, say $G_0 \uparrow$



equ. cannot move away from there because Y_0 is fixed

note: $Y_0 = z K_0^{\alpha} \underbrace{N_0}^{1-\alpha}$

if we reintroduce labour supply decisions $\uparrow G_0$ can $\uparrow Y_0$ and get us to point A

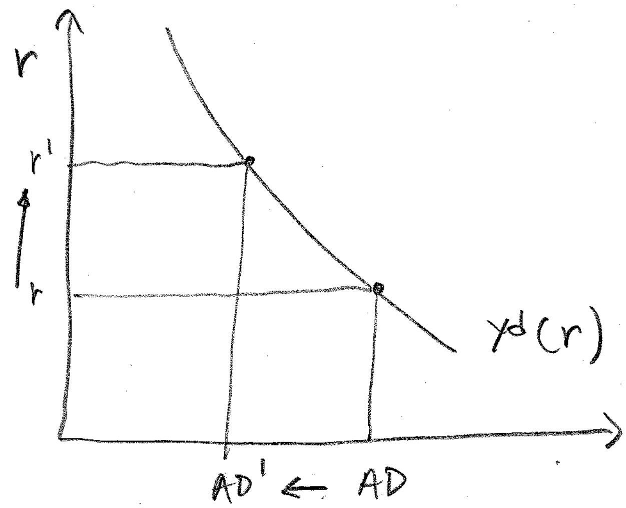
point A is not possible here because Y_0 is fixed

Ultimately, we will use this graph to draw or infer a relationship between the aggregate demand and r. We denote aggregate demand as a function of r by $Y^d(r)$

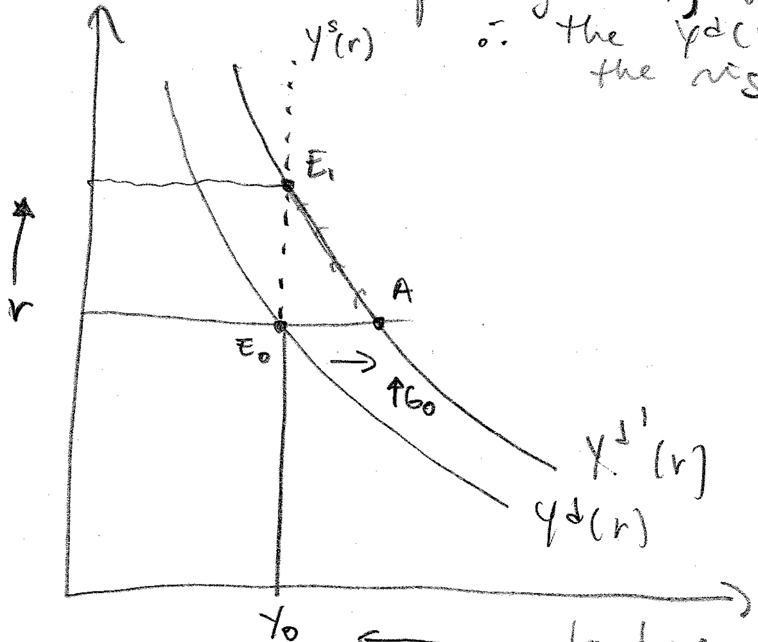
→ we know that $\uparrow r \rightarrow \downarrow AD$

∴ there must be an inverse relationship between the two

ie:



now, say $G_0 \uparrow$ [from previous page, we know that for a given r, we have $\uparrow AD$ [point A] ∴ the $Y^d(r)$ curves shifts to the right]



once we determine $Y^s(r)$, this will be different here $Y^s(r) = Y_0$

← output cannot ↑ here

∴ r must ↑ to reduce C_0 and restore the equ. at E_1