## Ricardian Equivalence Theorem

Given $G_0$ and $G_1$, the timing of taxes does not matter

↳ the equilibrium values of $r$, $C_0$ and $C_1$ are the same regardless of the precise values of $T_0$ and $T_1$ chosen, as long as the government's budget constraint is satisfied

→ Hence, the value of $B_0$ is irrelevant for finding the equilibrium. Consumers choose the same $C_0$ and $C_1$ whether $B_0 > 0$ or $B_0 = 0$.

↳ $G_0 = T_0$, $G_1 = T_1$

### Example

- $M$ identical consumers
  - same income $y_0, y_1$
  - same preferences
  - same taxes $t_0, t_1$
- $u = \ln(c_0) + \beta \ln(c_1)$
- government's goal: spend $G_0$ and $G_1$

↳ taxes are the same: $\Rightarrow T_0 = M t_0$, $T_1 = M t_1$

## Competitive Equilibrium

1) given r, $MRS^i_{c_0 c_1} = 1+r$

$$c_0^i + \frac{1}{1+r} c_1^i = y_0^i - t_0^i + \frac{1}{1+r}(y_1^i - t_1^i)$$

must be true for all M consumers

$\Rightarrow$ will give us $S_0^P(r)$

2) given r, $T_0 + \frac{1}{1+r} T_1 = G_0 + \frac{1}{1+r} G_1$

$\hookrightarrow$ the choice of $T_0$ and $T_1$ implies a value for $B_0$, given r. (ie: $B_0(r)$)

$\Rightarrow$ will give us $S_0^g(r) = -B_0(r)$

3) Equilibrium: $r$ is such that $S_0^P(r) + S_0^G(r) = 0$

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consumer $i = 1, 2, \ldots M$

1) $$MRS_{c_0 c_1} = \frac{\frac{\partial u}{\partial c_0}}{\frac{\partial u}{\partial c_1}} = \frac{\frac{1}{c_0^i}}{\beta \frac{1}{c_1^i}} = \frac{c_1^i}{\beta c_0^i}$$

$$\boxed{\frac{c_1^i}{\beta c_0^i} = 1+r} \quad \forall i$$

$$\boxed{c_0^i + \frac{1}{1+r} c_1^i = y_0^i - t_0^i + \frac{1}{1+r}[y_1^i - t_1^i]}$$

note: identical consumers $\Rightarrow$ $c_0^i = c_0^* \ \forall i$
$c_1^i = c_1^* \ \forall i$

since $y_0^i = y_0 \quad t_0^i = t_0$
$y_1^i = y_1 \quad t_1^i = t_1 \quad \forall i$

Thus, in the aggregate

$$C_0 = \sum_{i=1}^{M} c_0^i = \sum_{i=1}^{M} c_0^* = M c_0^*$$

$$C_0 = M c_0^* \quad \text{or} \quad c_0^* = \frac{C_0}{M}$$

Similarly, $\quad C_1 = M c_1^* \quad \text{or} \quad c_1^* = \frac{C_1}{M}$

$$Y_0 = M y_0$$

$$Y_1 = M y_1$$

Then, the condition $\quad \dfrac{c_1^i}{\beta c_0^i} = 1+r$

can be rewritten as

$$\underbrace{\frac{M}{M}}_{=1} \times \frac{c_1^i}{\beta c_0^i} = 1+r$$

$$\frac{1.1 \; M c_1^*}{M c_0^*} = 1+r$$

$$\boxed{\frac{C_1}{\beta \underbrace{C_0}_{\text{aggregate}}} = 1+r}$$

and $\quad M c_0^i + \dfrac{M}{1+r} c_1^i = M[y_0^i - t_0^i] + \dfrac{M}{1+r}[y_1^i - t_1^i]$

is the same as

$$\boxed{C_0 + \frac{1}{1+r} C_1 = Y_0 - T_0 + \frac{1}{1+r}[Y_1 - T_1]}$$

in addition,

$$C_0 + \frac{1}{1+r} C_1 = Y_0 + \frac{1}{1+r} Y_1 - \underbrace{\left[T_0 + \frac{1}{1+r} T_1\right]}_{= G_0 + \frac{1}{1+r} G_1}$$

using 2) in the def. of the C.E.

$$\therefore \boxed{C_0 + \frac{1}{1+r} C_1 = Y_0 + \frac{1}{1+r} Y_1 - \left[G_0 + \frac{1}{1+r} G_1\right]}$$

As $\dfrac{C_1}{\beta C_0} = 1+r$, $C_1 = \beta(1+r) C_0$

$$\therefore C_0 + \frac{1}{1+r}\left[\beta(1+r) C_0\right] = Y_0 + \frac{1}{1+r} Y_1 - \left[G_0 + \frac{1}{1+r} G_1\right]$$

$C_0^*(r)$

$$\boxed{C_0^* = \frac{1}{1+\beta}\left[Y_0 + \frac{1}{1+r} Y_1 - \left[G_0 + \frac{1}{1+r} G_1\right]\right]}$$

note: $C_0^*$ does not vary with $T_0$ or $T_1$
it only depends on $G_0$ and $G_1$

note: $C_0^*$ is obtained using 1) and 2) given r. We only need to find r using 3) and this $C_0^*$

3) requires that $r$ be such that

$$S_0^P(r) + S_0^G(r) = 0$$

or

$$\underbrace{Y_0 - T_0 - C_0^*(r)} + \underbrace{(-B_0)} = 0$$

$$\therefore \quad Y_0 - C_0^*(r) - \underbrace{[T_0 + B_0]}_{= G_0} = 0$$

$= G_0 \rightarrow$ from government's budget constraint in period 0

$$Y_0 - C_0^*(r) - G_0 = 0$$

or

$$\boxed{Y_0 = C_0^*(r) + G_0}$$

$Y_0$: Aggregate supply of goods

$C_0^*(r) + G_0 \rightarrow$ aggregate demand

↳ Hence, finding $r$ just amounts to find $C_0^*$ which will "clear" the goods market

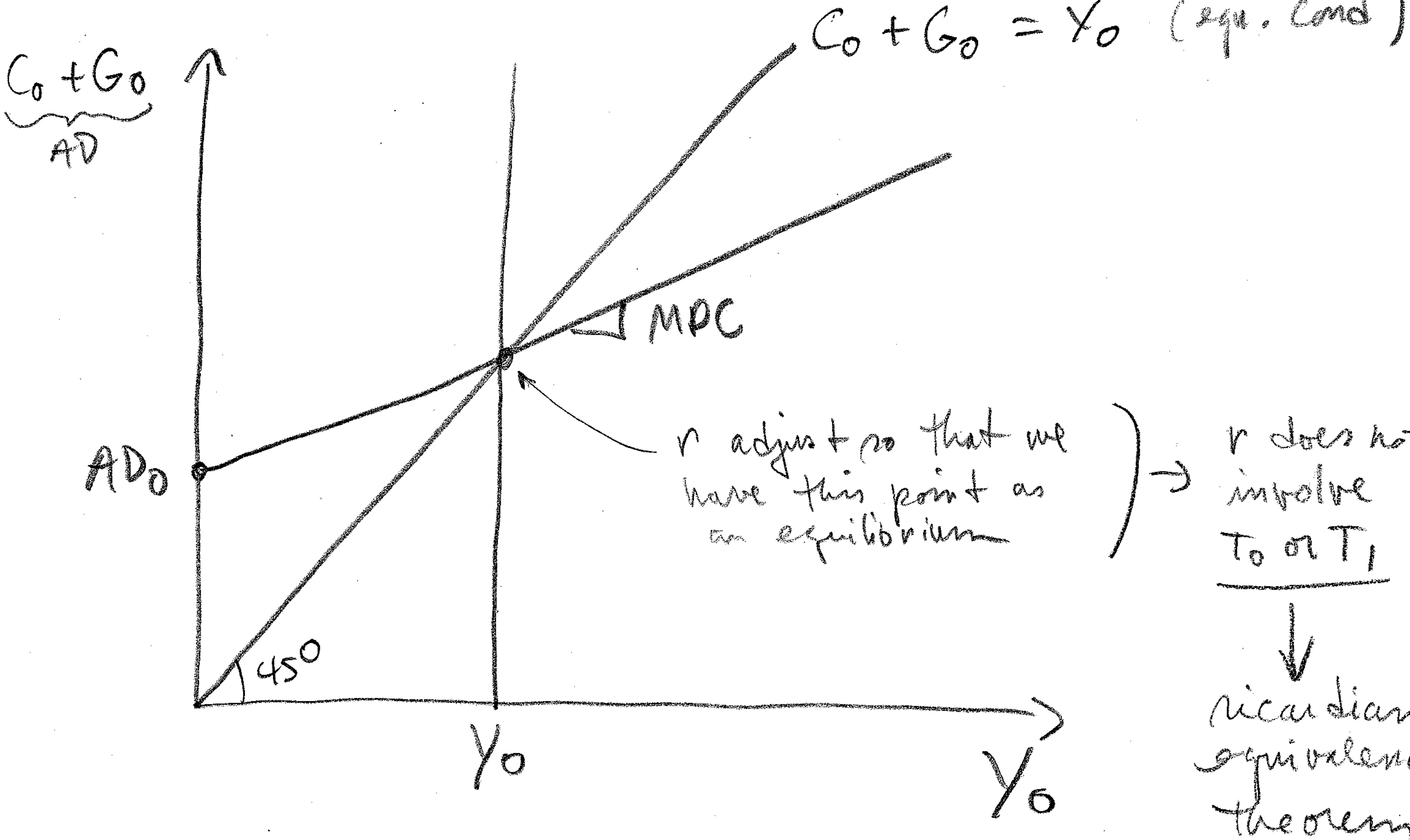
note: $Y_0 = C_0^*(r) + G_0$ <u>does not</u> involve $T_0$ or $T_1$

↳ $\therefore$ $r$ is independent of $T_0$ and $T_1$

$$AD(\underset{\ominus}{r}, \underset{\oplus}{Y_0}, \underset{\oplus}{Y_1}, \underset{\oplus}{G_0}, \underset{\ominus}{G_1}) = C_0^* + G_0$$

$$= \frac{1}{1+\beta}\left[Y_0 + \frac{1}{1+r}Y_1 - \left[G_0 + \frac{1}{1+r}G_1\right]\right] + G_0$$

$$= \underbrace{\frac{1}{1+\beta}}_{\to\ \text{MPC}} Y_0 + \underbrace{\frac{1}{(1+\beta)(1+r)}Y_1 + \frac{\beta}{1+\beta}G_0 - \frac{1}{(1+r)(1+\beta)}G_1}_{AD_0}$$




r adjust so that we have this point as an equilibrium → r does not involve $T_0$ or $T_1$

↓

ricardian equivalence theorem: whatever $T_0$ and $T_1$ are, r remains unchanged ($C_0$ and $C_1$ as well)

what do $T_0$ and $T_1$ change then? ⟸

ex: Say $Y_0 = 13$ $\quad G_0 = 3$ $\quad \beta = \frac{1}{1.1}$

$Y_1 = 11$ $\quad G_1 = 1$

Scenario B

$T_0 = G_0$

$T_1 = G_1$ $\Big\} \rightarrow$ which imply $B_0 = 0$

$S_0^P(r) = Y_0 - T_0 - C_0^*(r)$

$S_0^G(r) = 0$

$\therefore S_0^P(r) + S_0^G(r) = 0$

$Y_0 - G_0 - C_0^* + 0 = 0$

$C_0^* = 13 - 3 = 10$

$\frac{1.1}{2.1}\left[13 + \frac{1}{1+r}11 - \left[3 + \frac{1}{1+r}1\right]\right] = 10$

$\frac{1.1}{2.1}\left[10 + \frac{10}{1+r}\right] = 10$

$1.1\left[10 + \frac{10}{1+r}\right] = 21$

$11 + \frac{11}{1+r} = 21$

$\boxed{r^* = 0.10}$

note: $S_0^P(r) = 13 - 3 - 10 = 0$

Scenario B

$T_0 = 2$ (ie: $B_0 = 1$)

which requires

$T_1 = G_1 + (1+r) * 1$

$T_1 = 2 + r$

$S_0^P(r) + S_0^G(r) = 0$

$\Leftrightarrow Y_0 = C_0^* + G_0$

$\therefore \boxed{r^* = 0.10 \text{ too}}$

same $C_0^*$

note: $S_0^P(r) = 13 - 2 - 10$

$= 1$

ie: only saving changes