Econ 303 $\boxed{10}$

A two period model: → period 0 and period 1

↳ • we assume 2 periods : • people actually live longer but this is enough to get an idea of intertemporal issues [and much simpler]

↳ individuals make decisions for these two periods

• alternatively, we could see this as a period when old and another when young {individual decisions}

• we have M consumers / workers with the following preferences:

Preferences → over leisure and consumption

ie: $U(\underbrace{c_0, l_0}_{\text{1st period}}, \underbrace{c_1, l_1}_{\text{2nd period}})$

Assumptions

#0) workers supply labour inelastically

↳ ie: labour supply does not react to changes in wages (nor income)

$$\therefore \left.\begin{matrix} l_0 = \bar{l} \\ l_1 = \bar{l} \end{matrix}\right\} \rightarrow \text{constant}$$

$$\Rightarrow U(c_0, l_0, c_1, l_1) = U(c_0, \bar{l}, c_1, \bar{l}) = U(c_0, c_1)$$

(we will relax this assumption later on)

↳ not too important: we saw that it is employment that varies, not hours per worker

→ focus on c_0 and c_1 for now.

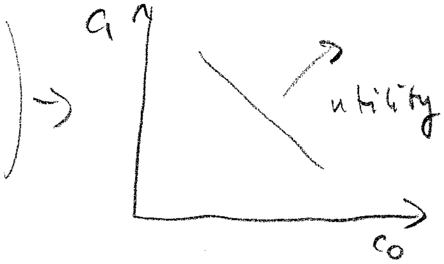
#1) Positive marginal utility [More is preferred to less]

↳ $\uparrow c_0$ and $\uparrow c_1$ always $\uparrow$ utility

i.e.: $MU_{c_0} > 0$

$MU_{c_1} > 0$ →

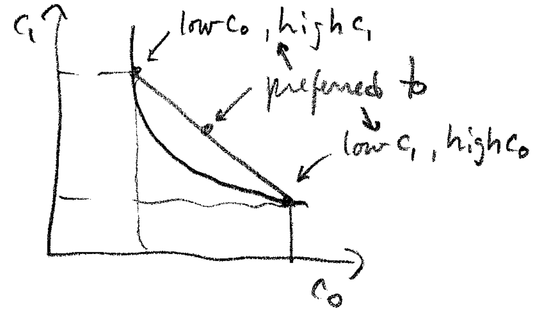
↳ $\frac{\partial u}{\partial c_1}$



#2) Decreasing marginal utility [preferences for diversity]

↳ $MU_{c_0} \downarrow$ as $c_0 \uparrow$

$MU_{c_1} \downarrow$ as $c_1 \uparrow$ →



#3) c_0 and c_1 are normal goods

↳ $\uparrow$ income → $\uparrow c_0$ and $\uparrow c_1$

↳ this is called consumption smoothing

i.e. consumer prefers to spread an $\uparrow$ in income over his lifetime rather than consuming it all right away

#4) c_0 is weakly preferred to c_1

↳ impatience: everything else equal, people will tend to prefer a dollar of consumption now rather than a \$ of consumption tomorrow.

ex: $u(c_0, c_1) = c_0^{1/2} + \beta c_1^{1/2}$ for $0 \le \beta \le 1$

$\beta < 1$ $c_0^{1/2}$ is preferred to $c_1^{1/2}$

Budget constraint

- disposable income (y^d)

$$y_0^d = w_0 N_0^s + \pi_0 - t_0$$
$$= w_0(h - l_0) + \pi_0 - t_0$$
$$= \underbrace{w_0 (\underbrace{h - \bar{l}}_{\text{constant}}) + \pi_0}_{\downarrow} - t_0$$

$$y_0 - t_0$$ (ie: y_0 Δ if w_0 or π_0 do)

Similarly,

$$y_1^d = y_1 - t_1$$

- consumers can transfer income intertemporally (across period 0 and 1) by saving or borrowing
  - ↳ saving: $\downarrow c_0$ to $\uparrow c_1$ (put money away for future)
  - borrowing: $\uparrow c_0$ by $\downarrow c_1$ (when debt is repaid)

- in this economy, we assume that consumers can trade in a bond market:
  - consumers can buy bonds : save
  - consumer can sell bonds (issue bonds) : borrow

  ↳ Bonds are directly exchanged in the market. There are no banks or other intermediaries

  - Bond is riskless : no default on borrowing
    ie: borrow 1\$ → pay 1\$ for sure

- interest rate is "r"

↳ ∴ lend 1\$ (save) in period 0 ↦ get 1\$ back in period 1 + r (reward for lending)

$\downarrow C_0$ by 1 → $\uparrow C_1$ by $1+r$

Borrow 1\$ (issue Bond) in period 1 ↦ repay a \$ in period 1 + r (cost of lending)

$\uparrow C_0$ by 1 $\downarrow C_1$ by $1+r$

- let S_0 be saving [ie: # of bonds [worth 1\$] bought by a consumer]

↳ ∴ $S_0 > 0$

Hence, $S_0 < 0$ → borrowing (issuing a bond)

Therefore, in period 0, the budget constraint is:

$$C_0 + S_0 = y_0^d$$

$$\boxed{C_0 + S_0 = y_0 - t_0}$$

(note: $S_0 < 0 \rightarrow C_0 = y_0 - t_0 \underbrace{- S_0}_{+}$

$\therefore C_0 > y_0 - t_0$)

in period 1

$$C_1 + S_1 = y_1^d + (1+r) S_0$$

$$C_1 + S_1 = y_1 - t_1 + (1+r) S_0$$

↳ $S_1^* = 0$ because there is no period 2!

- no need to save for the future

$\uparrow C_1$ but no $\uparrow C_2$

↳ actually strong incentives to have $S_1 < 0$ (ie borrow without ever paying back)

- but $S_1 < 0$ not possible because nobody will lend to this person

$\therefore$

$$\boxed{c_1 = y_1 - t_1 + (1+r)s_0}$$

↳ note: no decisions are actually taken in period 1: c_1^* is determined in period 0 (given s_0)!

note: $s_0 = y_0 - t_0 - c_0$ (using the budget constraint at time 0)

$$\therefore \quad c_1 = y_1 - t_1 + (1+r)s_0$$

$$c_1 = y_1 - t_1 + (1+r)[y_0 - t_0 - c_0)$$

$$\frac{c_1 + (1+r)c_0}{1+r} = \frac{y_1 - t_1 + (1+r)(y_0 - t_0)}{1+r}$$

$$\boxed{c_0 + \frac{1}{1+r}c_1 = y_0 - t_0 + \frac{1}{1+r}[y_1 - t_1]}$$

↳ this is called the intertemporal Budget constraint.

why?

$$p_{c_0}c_0 + p_{c_1}c_1 = p_{c_0}(y_0 - t_0) + p_{c_1}[y_1 - t_1]$$

$$c_0 + \frac{p_{c_1}}{p_{c_0}}c_1 = y_0 - t_0 + \frac{p_{c_1}}{p_{c_0}}[y_1 - t_1]$$

ie: $\frac{p_{c_1}}{p_{c_0}} = \frac{1}{1+r}$ = price of consumption at time 1 relative to that at time 0.

ie: to buy one unit of c_1, need to reduce c_0 by $\frac{1}{1+r} < 1$ (given how one can transfer income over time) note: $\frac{1}{1+r} c_1$ is how much of time 0 consumption c_1 is equivalent to. (in the present)

↳ $c_0 + \frac{1}{1+r} c_1$ = present value of consumption
= (lifetime consumption)

similarly, $y_0 - t_0 + \frac{1}{1+r}(y_1 - t_1)$ = present value of disposable income
= lifetime wealth

so the lifetime budget constraint says:

PV of lifetime consumption = PV of disposable income

$$c_0 + \frac{1}{1+r} c_1 = W$$ (wealth, not w)

ex: $r = 0.10$ (10%)

↳ $\frac{p_{c_1}}{p_{c_0}} = \frac{1}{1+r} = \frac{1}{1.1} \cong 0.91$

to buy 1\$ of c_1 cost 0.91\$ in period 0

graphically:

$$c_0 + \frac{1}{1+r} c_1 = W$$

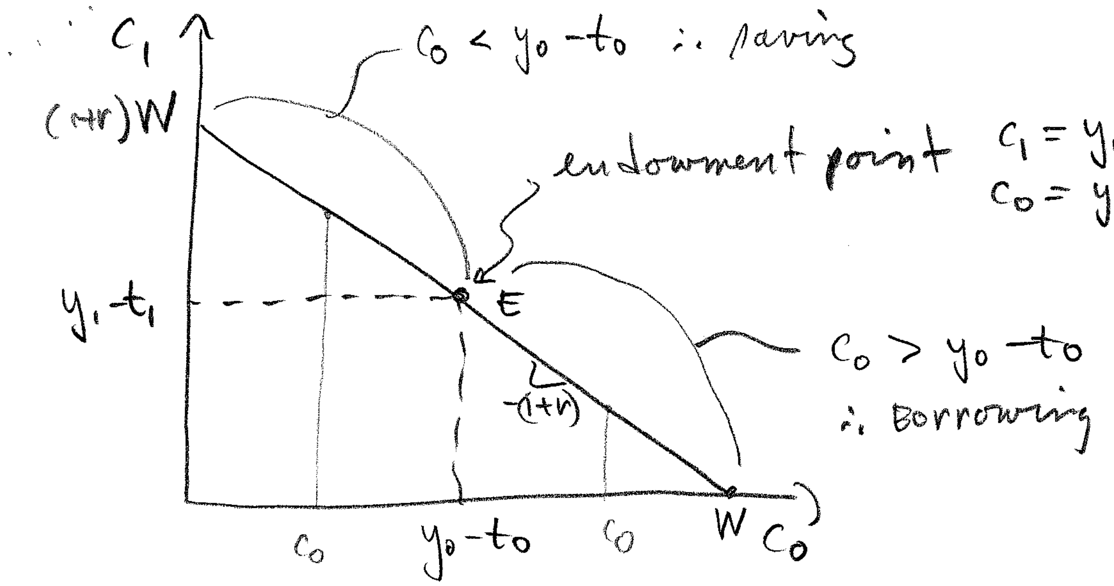
$$\frac{1}{1+r} c_1 = W - c_0$$

$$\boxed{c_1 = (1+r) W - (1+r) c_0}$$

ie: $c_1 = (1+r) W$ if $c_0 = 0$

$c_1 = 0$ if $c_0 = W$

$$\frac{\partial c_1}{\partial c_0} = -(1+r)$$



$c_1 = y_1 - t_1$
$c_0 = y_0 - t_0 \rightarrow s_0 = 0$
$\therefore (1+r)s_0 = 0$
$\therefore c_1 = y_1 - t_1$

Consumer's choice (optimal)

Problem: $\max_{c_0, c_1} \; u(c_0, c_1)$

s.t. $c_0 + \frac{1}{1+r} c_1 \leq W$ (given r)

value of consumption at c_0 in terms of c_1

solution: $MRS_{c_0 c_1} = \frac{p_{c_0}}{p_{c_1}}$ → cost of c_0 at c_1 $= 1+r$

rate of substitution c_1 for c_0

$c_0 + \frac{1}{1+r} c_1 = W$ (because $MU_{c_0} > 0$, $MU_{c_1} > 0$)

$\boxed{p_{c_0} c_0 + p_{c_1} c_1 = W}$

Again:

$MRS_{c_0 c_1} > \frac{p_{c_0}}{p_{c_1}}$ → value exceeds cost

$\therefore \uparrow c_0 \rightarrow \downarrow s_0$
(y^d_0 is fixed)
$\hookrightarrow \downarrow (1+r) s_0$
$\downarrow c_1$

$\rightarrow \frac{MU_{c_0} \downarrow}{MU_{c_1} \uparrow} \rightarrow \downarrow MRS_{c_0 c_1}$

$MRS_{c_0 c_1} < \frac{P_{c_0}}{P_{c_1}}$ $\Big| \rightarrow$ cost exceeds valuation

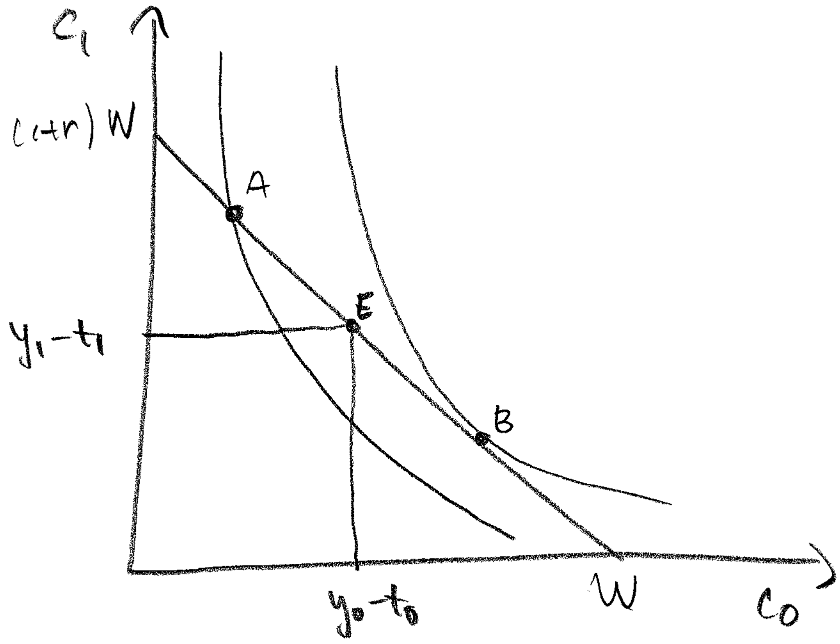
$\therefore \downarrow c_0 \rightarrow \uparrow s_0$

$\hookrightarrow \uparrow (1+r) s_0$

$\rightarrow \uparrow c_1$

$\therefore \frac{MU_{c_0} \uparrow}{MU_{c_1} \downarrow} \Big) \rightarrow \uparrow MRS_{c_0 c_1}$

$\therefore \quad MRS_{c_0 c_1} = \frac{P_{c_0}}{P_{c_1}} = 1+r \quad \left(\text{since } \frac{P_{c_1}}{P_{c_0}} = \frac{1}{1+r}\right)$



at A: $MRS_{c_0 c_1} > 1+r \quad \therefore \uparrow c_0 \downarrow c_1$

B: optimal $MRS_{c_0 c_1} = 1+r$

$c_1 = (1+r)W - c_0$

$\hookrightarrow$ note: $c_0^* > y_0 - t_0 \rightarrow$ borrower
(i.e. to the right of endowment point E)

Example:

$$u = c_0^{1/2} + \beta c_1^{1/2}$$

$$MRS_{c_0 c_1} = \frac{MU_{c_0}}{MU_{c_1}} = \frac{\frac{1}{2} c_0^{-1/2}}{\frac{\beta}{2} c_1^{-1/2}} = \frac{1}{\beta}\left(\frac{c_1}{c_0}\right)^{1/2}$$

optimality $\rightarrow$ $\frac{1}{\beta}\left(\frac{c_1}{c_0}\right)^{1/2} = 1+r$

$$\frac{c_1}{c_0} = [\beta(1+r)]^2$$

$$c_1^* = [\beta(1+r)]^2 c_0^*$$

feasibility $\rightarrow$ $c_0^* + \left(\frac{1}{1+r}\right) c_1^* = W$

$$c_0^* + \left(\frac{1}{1+r}\right)[\beta^2 (1+r)^2] c_0^* = W$$

$$c_0^* [1 + \beta^2 (1+r)] = W$$

$$c_0^* [1 + \beta^2 (1+r)] = W$$

$$\boxed{c_0^* = \underbrace{\left[\frac{1}{1+\beta^2(1+r)}\right]}_{<1} W}$$

$\beta = 0 \rightarrow c_0^* = W$

$\beta_1 = 1 \rightarrow c_0^* = \frac{1+r}{2+r}$

↳ ie: people consume a fixed fraction of lifetime income (wealth) [if r is fixed]

$$c_1^* = [\beta(1+r)]^2 \left(\frac{1}{1+\beta^2(1+r)}\right) W$$