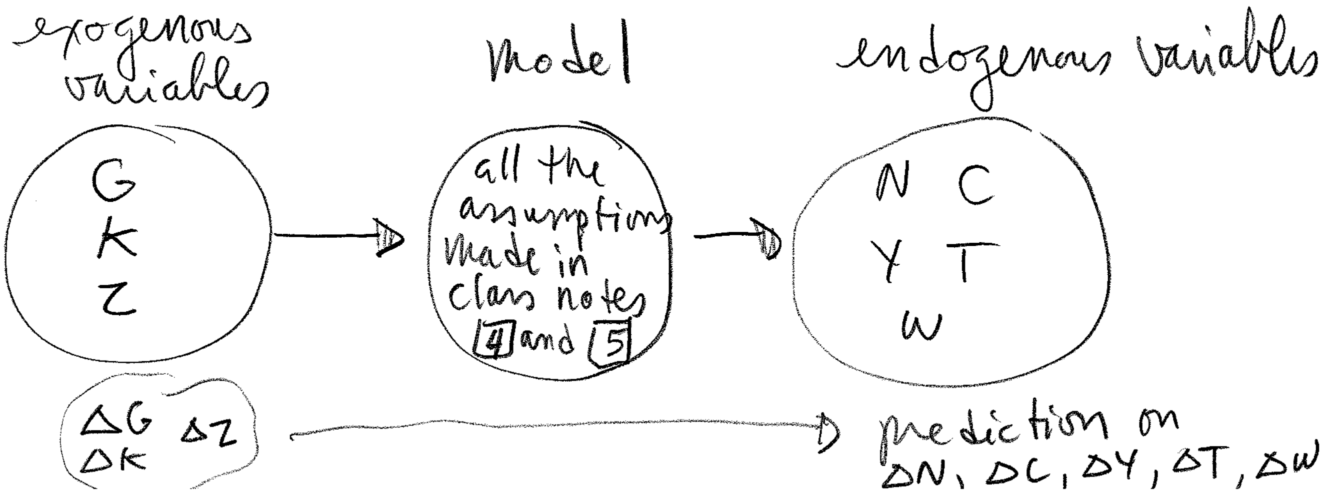
Equilibrium

→ given the behaviour of all agents in the economy [workers, firms, and the government], we can determine the competitive equilibrium of this economy.

→ Competitive equilibrium: equilibrium in which all agents take prices as given when they make their decisions. (ie: price-taking assumption)

→ here, the goal is to find the equilibrium values of N, C, Y, T, and w (for $P=1$) given G, z and K

ie:



Definition of a competitive Equilibrium

A competitive equilibrium is a wage level "w" and a price of goods "p" such that:

1) Given w, p, T, and π^e (expected profits), workers choose C and l to maximize utility $u(C, l)$ subject to the budget constraint $pC + wl = wh + \pi^e - T$

   ↳ workers do the best they can

2) Given w, p, K, and z, the firm chooses N^d to maximize profits $\pi = pY - wN^d$ where $Y = zK^\alpha (N^d)^{1-\alpha}$.

   ↳ Firms do the best they can.

3) The government's budget constraint is satisfied: $T = p \cdot G$

4) Markets clear:
   - w is such that $N^d = N^s = h - l = N$
   - p is such that $C + G = Y$

5) $\pi^e = \pi$: Rational expectations

   ↳ the profits used by consumers correspond to actual profits note : in condition 3), we can only use $N^d = N^s$ because we know from Walras law that $C + G = Y$ if it is the case.

↳ we can actually verify this :

from the consumers' budget constraint

$$pC + wl = wh + \pi^e - T$$

$$pC = w(h - l) + \pi^e - T$$

$$\boxed{pC = wN^s + \pi^e - T}$$

using $\pi^e = \pi = pY - wN^d$

and $T = pG$ in this constraint :

$$pC = wN^s + pY - wN^d - pG$$

$$\boxed{pC + pG = pY + w(N^s - N^d)}$$

note: $N^s = N^d \Rightarrow N^s - N^d = 0$

$$\therefore \cancel{p}C + \cancel{p}G = \cancel{p}Y$$

$$C + G = Y \quad \text{for any "p"}$$

ie: can set p to any value. $p = 1$ is convenient.

Solving the equilibrium: (with $p=1$)

1), 5) $\Rightarrow$ $w = MRS_{lc}$
$c + wl = wh + \pi - T$

2) $\Rightarrow$ $w = MPN$ $(\pi = \alpha Y)$

3) $\Rightarrow$ $T = G$

4) $\Rightarrow$ $N^d = h - l = N$

Together, we have to solve

$$\boxed{\begin{aligned} MRS_{lc} &= MPN \\ c + wl &= wh + \pi - T \\ \pi &= \alpha Y \end{aligned}}$$

for c^* and l^*.

↳ Doing so is not as easy as before (ie: finding the highest indifference curve given the budget constraint)

To see this, notice that:

1) $w = MPN = (1-\alpha) z K^{\alpha} N^{-\alpha}$

$$= (1-\alpha) z K^{\alpha} (h-\ell)^{-\alpha} = w(\ell)$$

in equilibrium

$\therefore \uparrow \ell \rightarrow \uparrow w$

ie: the slope of the budget constraint changes as ℓ is varied

2) $\pi = \alpha z K^{\alpha} N^{1-\alpha} = \alpha z K^{\alpha} (h-\ell)^{1-\alpha}$

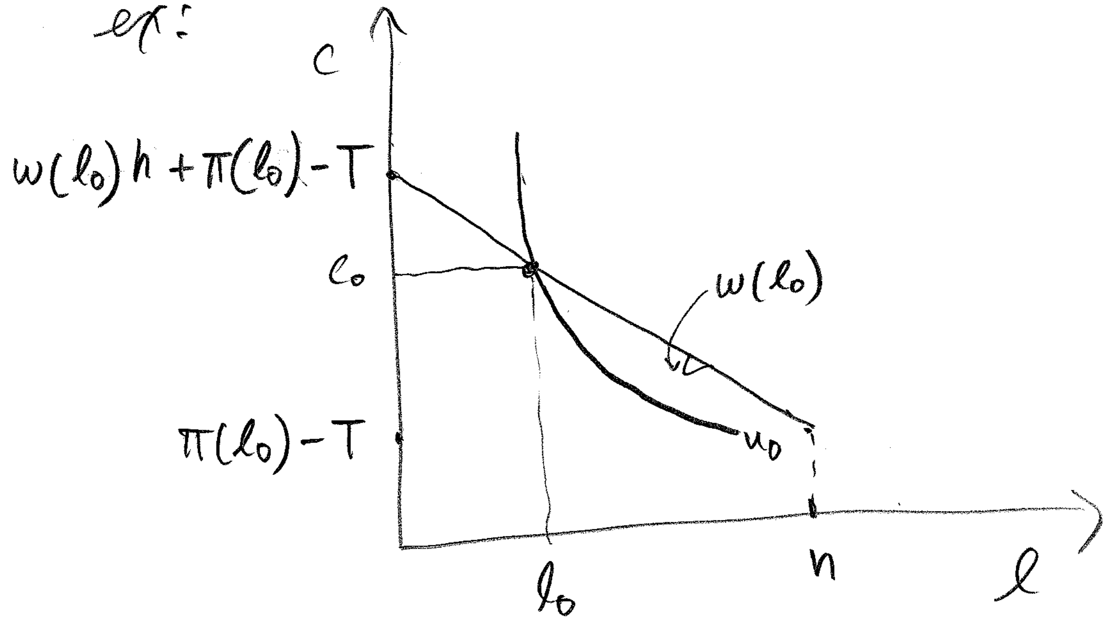
$$= \pi(\ell)$$

$\therefore \uparrow \ell \rightarrow \downarrow \pi$

ie: the intercept of the budget constraint is also changing as ℓ is varied

$\Rightarrow$ as ℓ is varied, the budget constraint changes so it is difficult to find the optimal point on a graph.

ex:



given l_0, we have the budget constraint above implied by $w(l_0)$ and $\pi(l_0)$

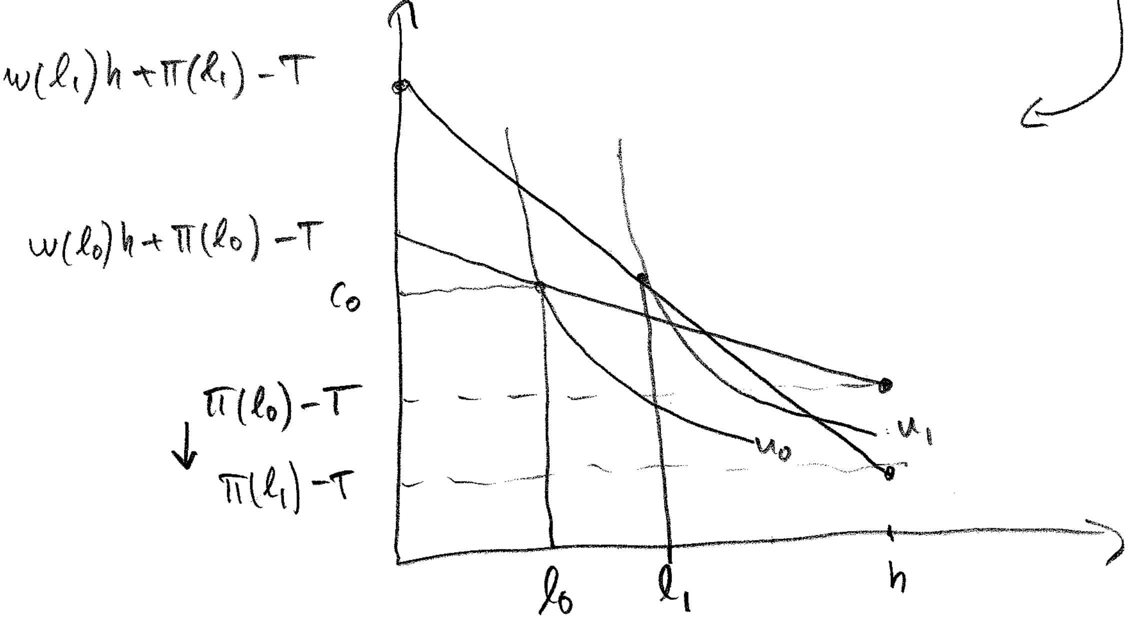
↳ at c_0, l_0, $MRS_{lc} > w$

∴ value of leisure > cost

$\Rightarrow \uparrow l$ from l_0 to l_1 (pay)

with l_1, we have (note: $w(l_1) > w(l_0)$, $\pi(l_1) < \pi(l_0)$)

budget constraint is moving when l moves from l_0 to l_1



There is actually an easier way to find the equilibrium.

↳ we saw that when $N^s = N^d$, the budget constraint became

$$C + G = Y$$

$$\therefore \quad C = Y - G$$

$$\boxed{C = zK^{\alpha}(h - \ell)^{1-\alpha} - G}$$

← because $N^d = N^s = h - \ell$

↳ note that this relationship gives us all the possible combinations of C and ℓ that they economy can "produce" given $\underline{G,\ z,\ \text{and } K}$

↳ all the exogenous variables

This relationship is called the production possibilities frontier (PPF)

It turns out that we can get the <u>competitive equilibrium</u> solution by solving

$$\max_{C,\ell} \; U(C, \ell)$$

subject to $C = zK^{\alpha}(h - \ell)^{1-\alpha} - G$

in this problem;

1) the value of leisure is MRS_{lc}

2) the cost of leisure is a decrease in Y

ie: $Y = zK^{\alpha}(h-l)^{1-\alpha}$

$$\frac{\partial Y}{\partial l} = -(1-\alpha)\, zK^{\alpha}(h-l)^{-\alpha}$$

$$= -MPN$$

ie income falls by the MPN

1) ∧ 2) $\Rightarrow$ $\uparrow l$ $\downarrow l$ until value = cost

$$\boxed{MRS_{lc} = MPN}$$

same condition that in page 4

$$\boxed{C = zK^{\alpha}(h-l)^{1-\alpha} - G}$$

note: $C = \underbrace{wN + \pi}_{Y} - T$

$$\boxed{C = Y - G}$$

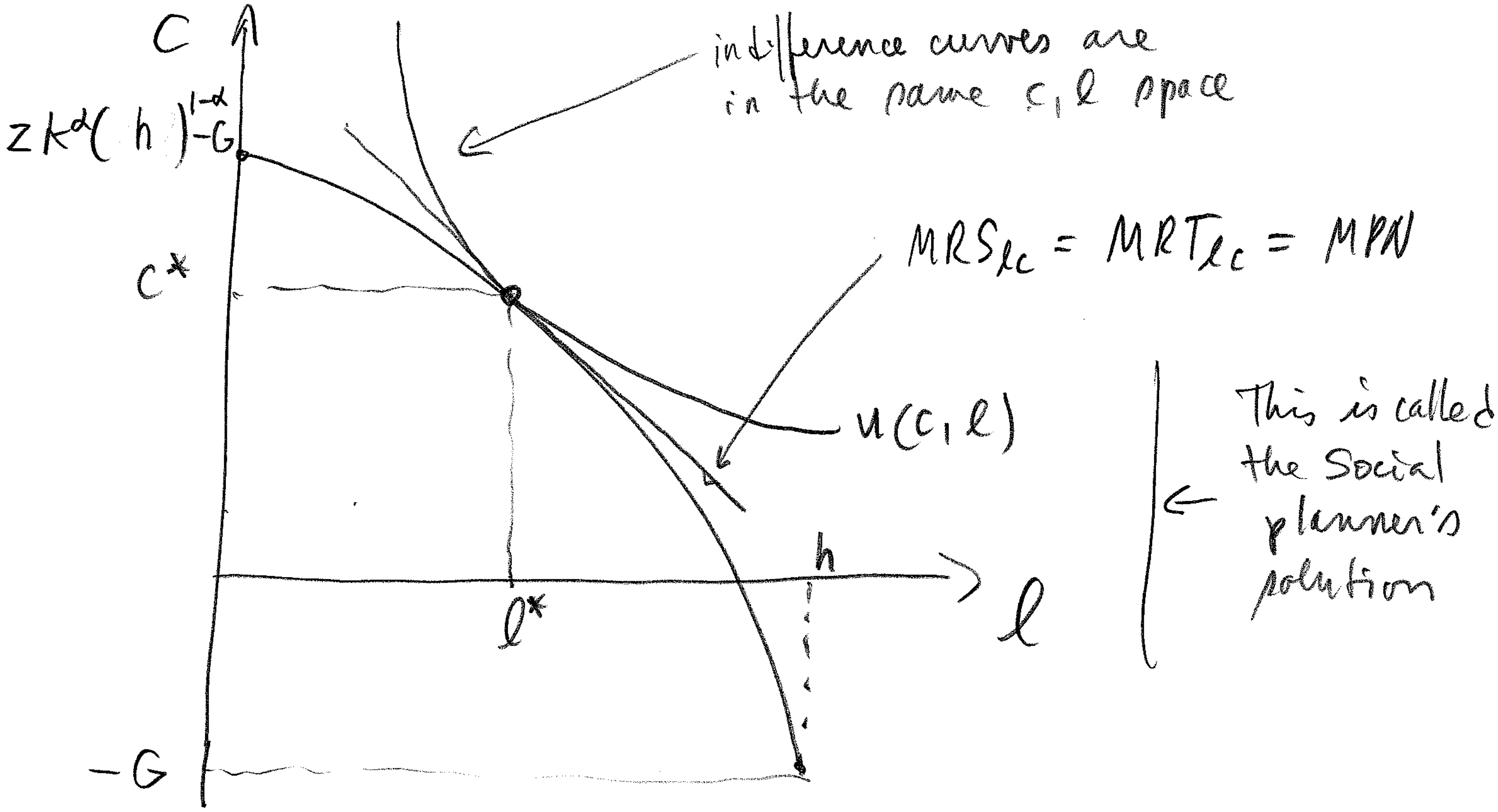
$\pi = Y - wN$

$\hookrightarrow \pi + wN = Y$

slope = MPN = MRT_{lc}

$\hookrightarrow$ marginal rate of transformation

rate at which the economy can transform l into C graphically, we have



note: when $l=0$, $C = zK^{\alpha}h^{1-\alpha} - G$

$l=h$, $C = 0 - G = -G$

Example: $u = c^{2/3} l^{1/3}$

$Y = zK^{1/3} N^{2/3}$

$G = 0$

$K=1$, $z=1$, $h=14$

with those, the production possibilities frontier is

$C = zK^{1/3} N^{2/3} - G$

$\boxed{C = (14-l)^{2/3}}$ since $N = h - l$

$$MPN = \frac{\partial Y}{\partial N} = \tfrac{2}{3} N^{-1/3}$$

$$\therefore \boxed{MPN = \tfrac{2}{3}(14-\ell)^{-1/3}}$$

$$MRS_{\ell c} = \frac{\frac{\partial u}{\partial \ell}}{\frac{\partial u}{\partial c}} = \frac{\tfrac{1}{3} c^{2/3} \cdot \ell^{-2/3}}{\tfrac{2}{3} c^{-1/3} \ell^{1/3}}$$

$$\therefore \boxed{MRS_{\ell c} = \frac{1}{2}\frac{c}{\ell}}$$

Planner's solution: $MRS_{\ell c} = MPN$

or $\boxed{\frac{1}{2}\frac{c}{\ell} = \tfrac{2}{3}(14-\ell)^{-1/3}}$

and $\boxed{c = (14-\ell)^{2/3}}$

Together: (replace "c" in the 1st equation)

$$\frac{1}{2}\frac{(14-\ell)^{2/3}}{\ell} = \frac{2}{3}(14-\ell)^{-1/3}$$

$$14-\ell = \frac{4}{3}\ell$$

$$14 = \left(\frac{4}{3}+1\right)\ell = \frac{7}{3}\ell$$

$$\therefore \ell = \frac{3}{7}\cdot 14$$

$\boxed{\ell = 6}$ or $N = 14-6$ $\boxed{N=8}$

(and $c = 8^{2/3} = 4$) $\boxed{c=4}$

Note:

$$MPN = MRS_{\ell c} = \frac{1}{2}\frac{4}{6}$$

$$\boxed{MPN = \frac{1}{3}}$$

we can check that this is also a competitive equilibrium

↳ ie: that $\ell = 6$ and $c = 4$ is a competitive equilibrium when $p = 1$ and $w = 1/3$ (MPN)

→ to do this, all we have to do is to check that the conditions 1-5 are satisfied

with $w = 1/3$ and $N = 8$

$$\pi = N^{2/3} - w \cdot N$$
$$= 8^{2/3} - \frac{1}{3} 8$$
$$\pi = 4 - \frac{8}{3}$$
$$\boxed{\pi = \frac{4}{3}}$$

with $\pi^e = \pi = 4/3$, the consumer's budget constraint is

$$c + \frac{1}{3}\ell = \frac{1}{3} \cdot 14 + \frac{4}{3} - 0$$

$$c + \frac{1}{3}\ell = \frac{18}{3} = 6$$

$$\underbrace{4 + \frac{1}{3} 6}_{c + w\ell} \text{ "=" } \underbrace{6}_{wh + \pi - T}$$

Budget constraint is satisfied by $c = 4$ and $\ell = 6$ when $w = 1/3$ and $\pi^e = \frac{4}{3}$

Moreover

$$MRS_{\ell c} = w$$
$$\frac{1}{2} \frac{c}{\ell} = w$$
$$\frac{1}{2} \frac{4}{6} = 1/3$$

→ optimality condition is also satisfied

⇒ This is also a competitive equilibrium we can illustrate this on a graph

↳ ie: competitive equilibrium = planner's solution