Firms

↳ firms are essentially described by the production function that they own:

$$Y = z\,F(K, N^d)$$

- Y → output (# of goods produced)
- z → total factor productivity (index of how productive machine and workers are)
- K → capital (machines)
- N^d → labour demand (# of hours)

→ Firms own the capital stock and hire labour to operate the production function.

→ Firms are owned by consumers
∴ ultimately, profits go back to consumers

→ Capital is fixed (K) → This is true in the short-run because it takes time to install capital

→ when there are several periods, capital varies over time with investment

→ total factor productivity (z)
↳ reflects the efficiency of the production process

→ productivity may fluctuate due to various factors:
- weather (farming)
- price of energy
- innovation
- regulations

↳ anything that allows a firm to produce more (or less) with the same quantity of inputs is reflected in productivity (z)

→ A key concept when we look at production decisions is the marginal product

↳ how much more output can an additional unit of input can produce (similar concept as marginal utility but applied to production)

value of labour → | MPN = marginal product of labour

↳ increase in output when labour is increased by one unit. (at the margin)

value of capital → | MPK = marginal product of capital

↳ increase in output when capital is increased by one unit (at the margin)

Assumptions regarding the production function

1) Constant returns to scale

↳ change in output when the scale of production is increased (all inputs are increased in the same proportion)

ex: compare the output of one factory with 100 to that of 2 factories with 200 workers [when the scale of production is doubled]. Suppose that one factory with 100 workers produces 50 000 units of output.

↳ ie: $K=1$ and $N=100 \Rightarrow Y=50000$

The question is: when the scale of production doubles (from $K=1$ and $N=100$ to $K=2$ to $N=200$), does output doubles or increases by less or more?

a) if $Y = 100\,000$ (doubles)
↳ this is constant returns to scale

learning by doing ← b) if $Y = 110\,000$ (more than double)
↳ this is increasing returns to scale

managers for firm production ← c) if $Y = 90\,000$ (less than doubles)
↳ this is decreasing returns to scale

⇒ Here, we assume constant returns to scale

↳ why? in this case, the number of firms do not affect the scale of production

i.e.: 2 firms with 100 worker each produce the same amount as one firm with 200 workers

i.e.: $\boxed{\begin{array}{ll} K=1 & N=100 \\ K=1 & N=100 \end{array}} = \boxed{\begin{array}{l} K=2 \\ N=200 \end{array}}$

↳ This means that if we solve for the optimal decisions of one large firm with all the economy's capital stock and that employs all available workers, we get the same solution as an economy with several smaller firms

2) $MPN > 0 \qquad MPK > 0$

(i.e: positive marginal products)

↳ $\uparrow K$ or $\uparrow N$ always $\uparrow Y$

more inputs always lead to more output

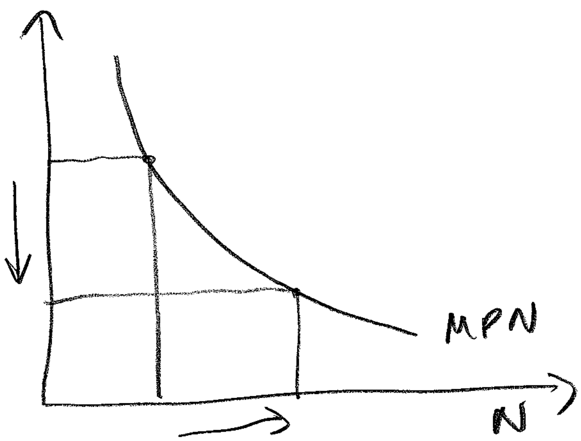
3) decreasing marginal products

↳ congestion effect

$\uparrow N \longrightarrow \downarrow MPN$

$\uparrow K \longrightarrow \downarrow MPK$

↳ means that the productivity of an additional worker (given a fixed number of machines) falls as more workers are hired



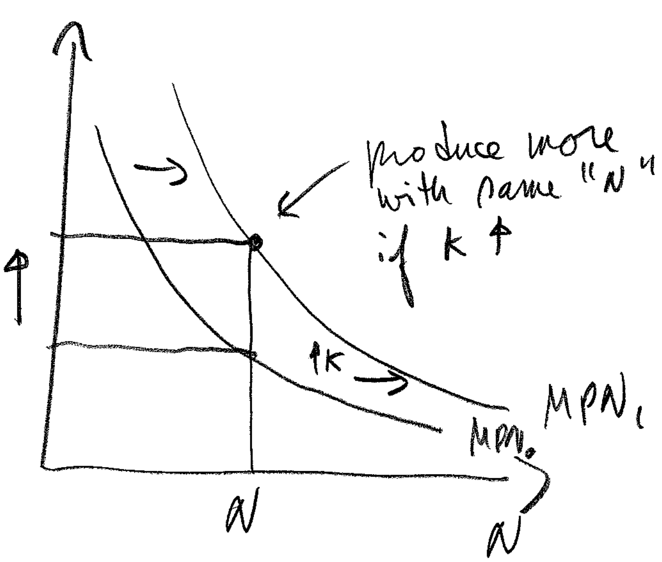
4) $MPN \uparrow$ as $K \uparrow$

$MPK \uparrow$ as $N \uparrow$

→ increasing one input increases the productivity of the other

↓

because it reduces the congestion effect

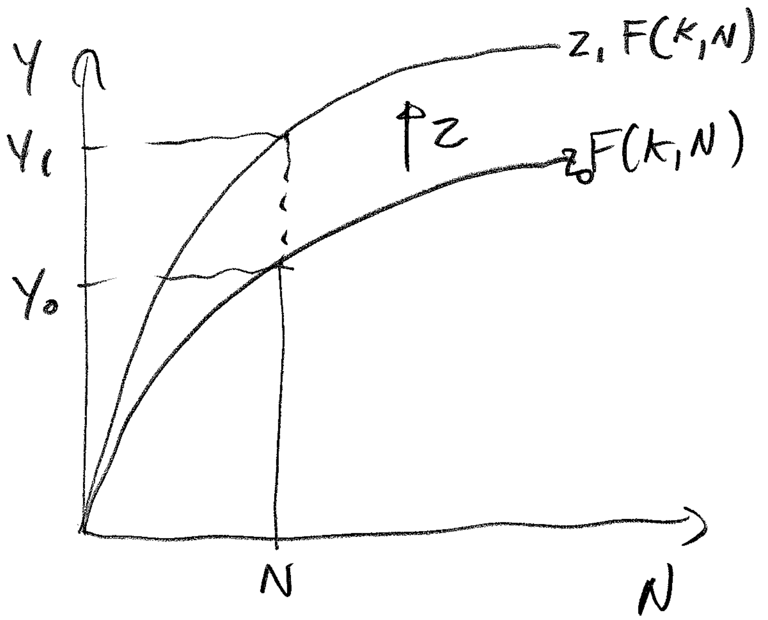


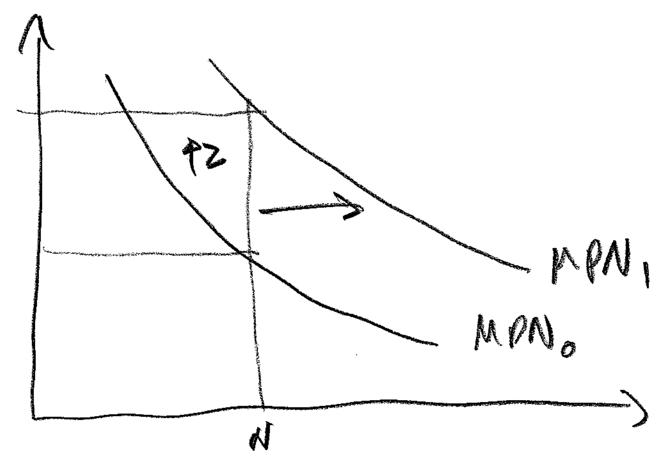
$MPN \downarrow$ as $N \uparrow$ because the capital stock is fixed. Adding more capital makes all workers more productive in the end, 3) and 4) imply that what really matters is the number of machines per worker

ie: $\frac{K}{N}$ = capital/labour ratio

note: another thing that improves productivity is an ↑ in z

↳ ↑z → ↑Y (holding K and N the same)





ex: Cobb-Douglas production function

$$\boxed{Y = z K^{\alpha} N^{1-\alpha}}$$

$$\boxed{\begin{aligned} MPN &= (1-\alpha)\, z \left(\frac{K}{N}\right)^{\alpha} \\ MPK &= \alpha\, z \left(\frac{K}{N}\right)^{\alpha-1} \end{aligned}}$$

2) $$MPN = \frac{\partial Y}{\partial N} = (1-\alpha)\, z K^{\alpha} N^{-\alpha} > 0 \quad (\text{since } N>0,\ K>0)$$

$$MPK = \frac{\partial Y}{\partial K} = \alpha z K^{\alpha-1} N^{1-\alpha} > 0$$

3) $\downarrow$ MPN as $N \uparrow$: $$\frac{\partial MPN}{\partial N} = \underbrace{-\alpha(1-\alpha)}_{<0}\, \underbrace{z K^{\alpha} N^{-\alpha-1}}_{>0}$$

$$< 0$$

4) MPN $\uparrow$ as $K \uparrow$: $$\frac{\partial MPN}{\partial K} = \underbrace{\alpha(1-\alpha)}_{+}\, \underbrace{z K^{\alpha-1} N^{-\alpha}}_{>0}$$

$$> 0$$

note: $$MPN = (1-\alpha)\, z K^{\alpha} N^{-\alpha}$$

$$\therefore\ N \cdot MPN = (1-\alpha)\, \underbrace{z K^{\alpha} N^{1-\alpha}}_{Y}$$

$$N \cdot MPN = (1-\alpha)\, Y$$

$$\therefore\ \boxed{MPN = (1-\alpha)\frac{Y}{N}}$$

same for MPK, $\boxed{MPK = \alpha \frac{Y}{K}}$

X $$\text{elasticity} = \frac{\partial Y}{\partial K} \cdot \frac{K}{Y} = MPK \cdot \frac{K}{Y} = \alpha \frac{Y}{K}\frac{K}{Y} = \alpha$$

profit maximization

π = sales − cost of production

$$\pi = zF(K, N^d) - wN^d$$

P=1

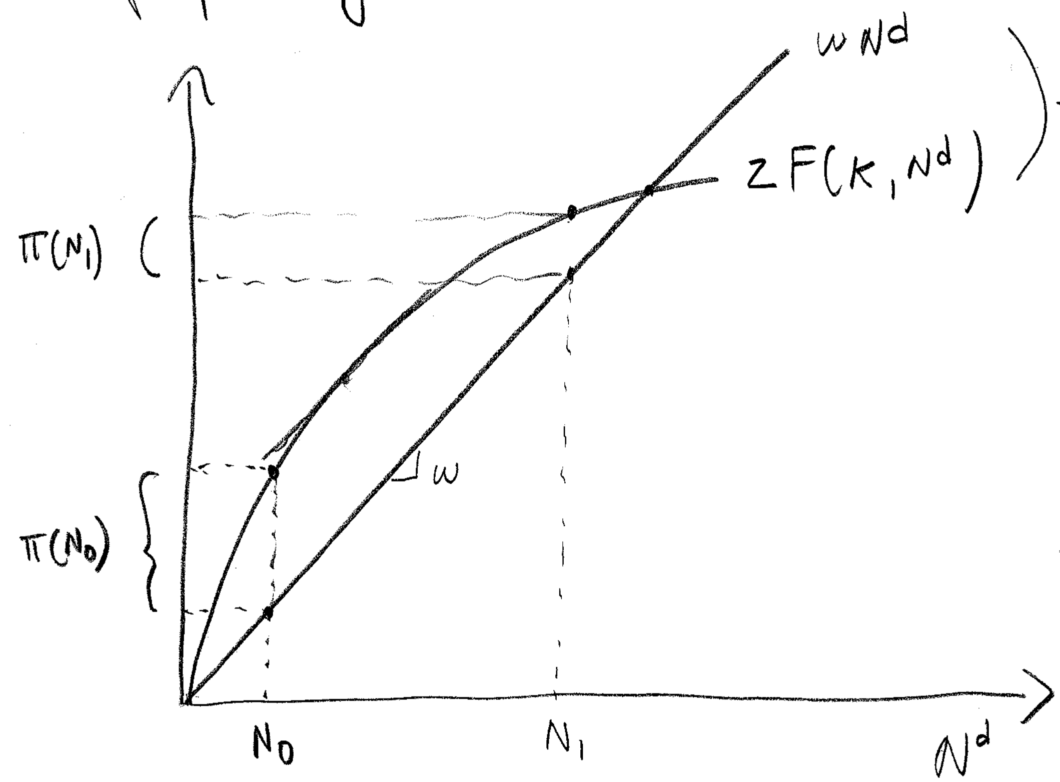
$\therefore$ the firm's problem is:

$$\max_{N^d} \; zF(K, N^d) - wN^d$$

(recall: K is fixed)

graphically:

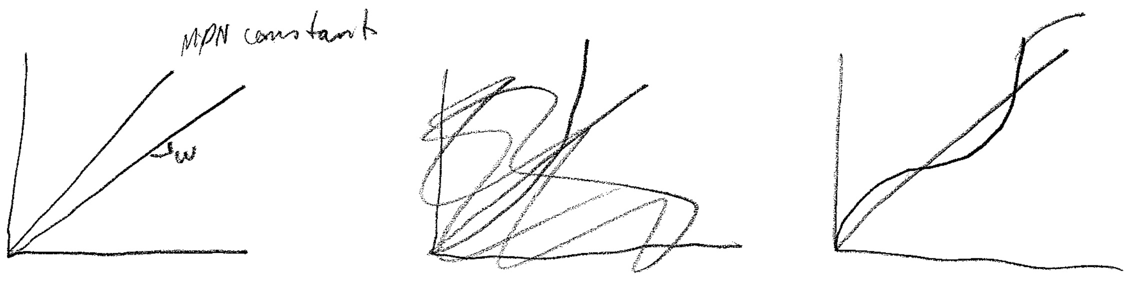
variable cost function



profit is the difference between these two curves

↓

need to maximize the distance between these two curves



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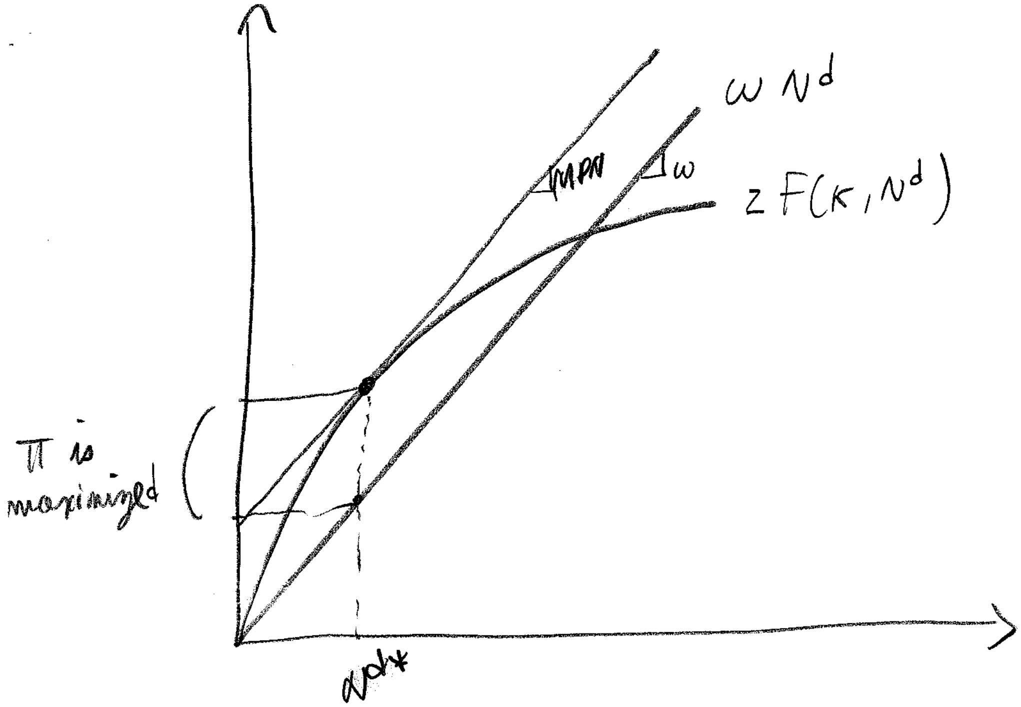
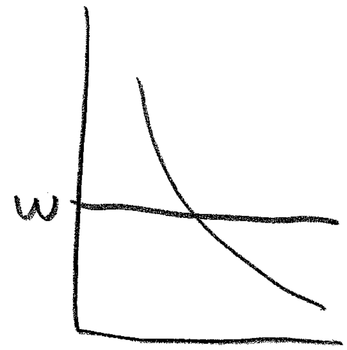




condition for profit maximization:

$$\boxed{MPN = w}$$

why?

1) say $MPN > w$

this means that ↑N brings a higher increase in output than the cost of additional workers

∴ benefit > cost ⟶ ↑N

that ↑ in N ⟶ ↓ MPN (Assumption #3)

[so MPN could still exceed w but now by a lesser amount]

this continues until there is no more incentives to do that

ie: when $MPN = w$

2) pay $MPN < w$

↳ $\uparrow N$ by 1 unit brings less benefits than the cost of $\uparrow N$ by 1 unit (w)

∴ benefit < cost $\longrightarrow \downarrow N$

that $\downarrow$ in $N \longrightarrow \uparrow MPN$

(assumption #3)

... and this continues until

$$w = MPN$$

$\Rightarrow$ $MPN = w$ is the only possible case.

note: $w = MPN = \frac{\partial Y}{\partial N}$

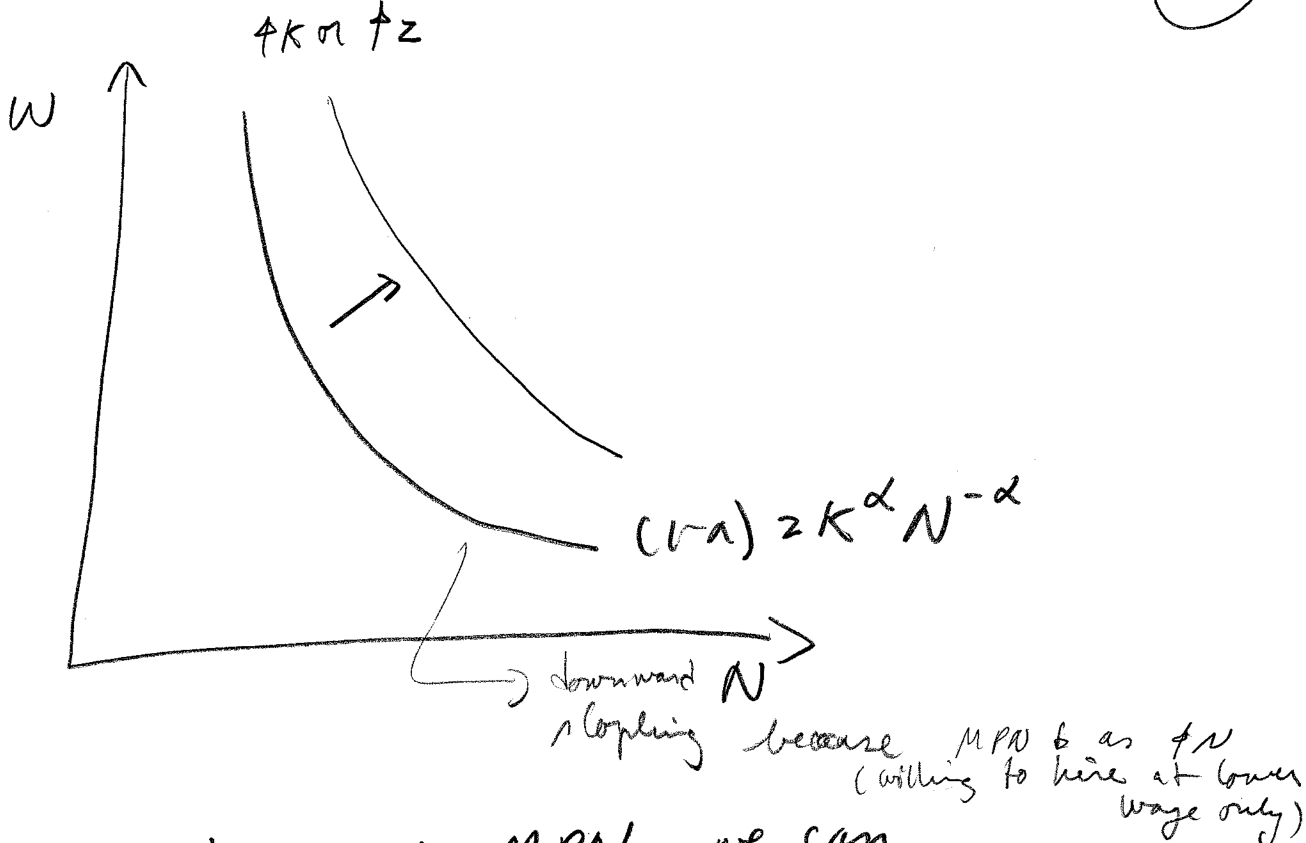
↳ this is a function of N

∴ this is our <u>inverse</u> labour demand curve

ex: $Y = zK^{\alpha}N^{1-\alpha}$

$$MPN = \frac{\partial Y}{\partial N} = (1-\alpha)zK^{\alpha}N^{-\alpha}$$

$$\therefore \boxed{w = (1-\alpha)zK^{\alpha}N^{-\alpha}}$$



note: given $w = MPN$, we can calculate profits

$$\pi^* = z\,F(K, N^d) - MPN \cdot N^d$$

with $Y = zK^\alpha N^{1-\alpha}$

$$\pi = zK^\alpha N^{1-\alpha} - (1-\alpha) zK^\alpha N^{-\alpha} \cdot N$$

$$= zK^\alpha N^{1-\alpha} - \underbrace{(1-\alpha) zK^\alpha N^{1-\alpha}}_{wN = (1-\alpha)Y}$$

$$\pi = \alpha zK^\alpha N^{1-\alpha}$$

$$\boxed{\pi = \alpha Y}$$

government

- The last thing we need to specify in our model is what the government does [recall that the consumers budget constraints involves taxes T]

- the government's budget constraint is:

$$\underbrace{G_t + r_t B_t}_{\text{spending}} = \underbrace{T_t + \underbrace{B_{t+1} - B_t}_{\text{increase in debt}}}_{\text{revenues}}$$

r_t: interest rate; B_t: government's debt; T_t: taxes

- if $T_t < G_t + r_t B_t$, revenues are too little compared to spending

  $\therefore$ need to borrow more to sustain spending

$\Rightarrow$ in a one period model, there cannot be any debt [nobody is saving, nobody will hold this debt]

$\therefore B_t = B_{t+1} = 0$

$$\Rightarrow \boxed{G_t = T_t} \quad \text{or} \quad \boxed{G = T}$$

ie: taxes are equal to total government spending [which is given (exogenous)]