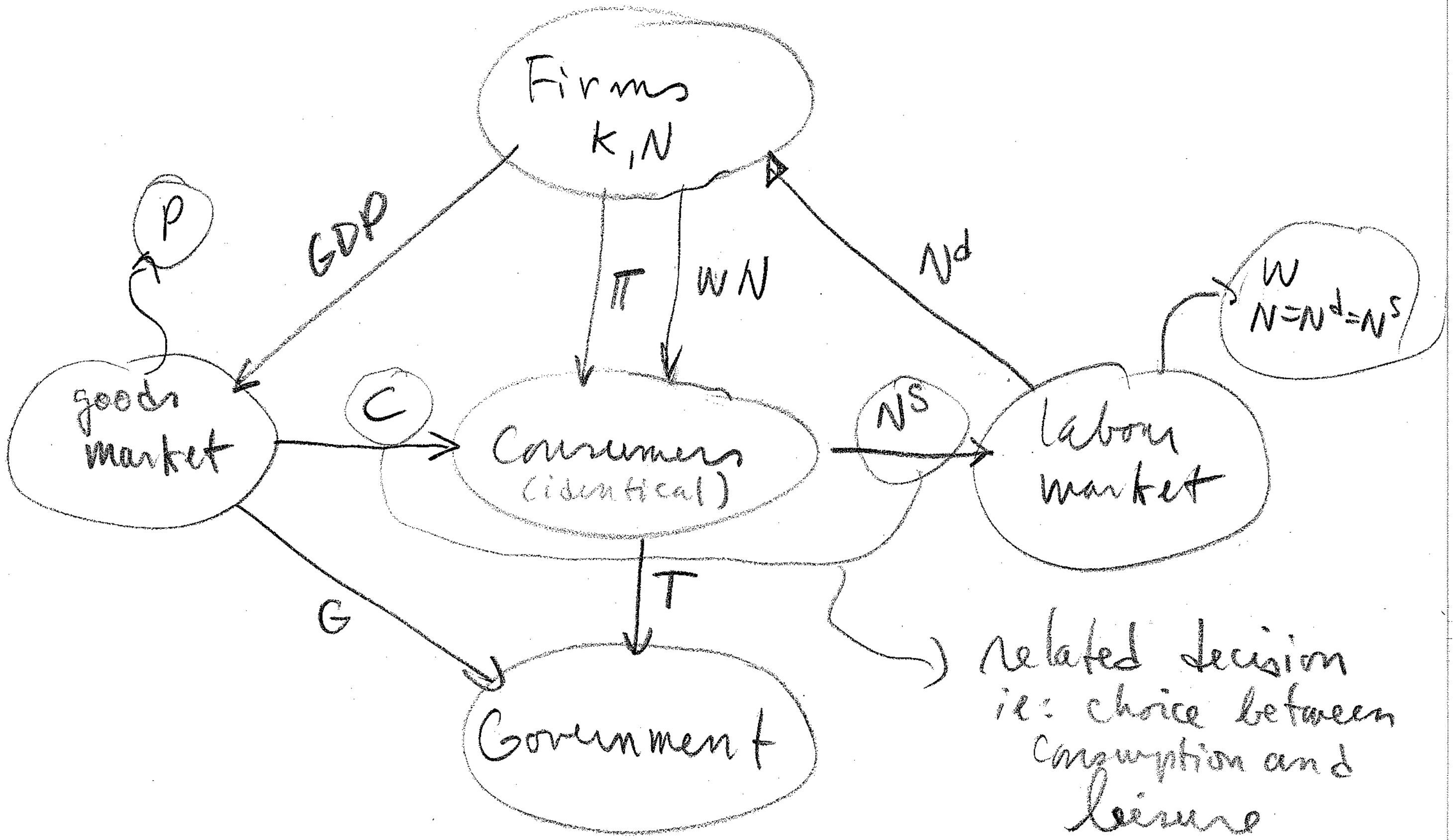
Econ 303 $\boxed{4}$

one period model



- $GDP = C + G$
- GDP = # of goods produced ← production function
- $GDP = \pi + wN$ (ie National income) $\Rightarrow Y^d = Y - T$
- Walras law → set $P = 1$

  ∴ implicitly, $w = \frac{W}{P}$ = real wage

  - $\frac{GDP}{P} = GDP$ = real GDP
- aggregation → identical consumers with possibly different income → only aggregate income matters

  ↳ one consumer

Consumer's problem $\mapsto$ · representative consumer · identical $\to$ aggregation

- Consumers are endowed with h units of time which can be spent working (labour supply) or enjoying leisure

  N^s = labour supply = time spent working

  ℓ = leisure time

  $\therefore$ time constraint is $N^s + \ell = h$

  trade-off: $\uparrow N^s$ requires $\downarrow$ in ℓ (or vice versa)

- consumers value leisure (and thus dislike work) and consumption

  ↳ we say that consumers "consume" consumption goods and leisure

  - a consumption bundle (c, ℓ) is valued using the utility function $u(c, \ell)$
  - This utility function satisfies all 3 assumptions made before

- When they supply labour, workers receive the wage rate w for each unit of time supplied
- Consumers also receive firms' profits (they own firms) and pay taxes (lump sum) to the government

$$\therefore \quad Y^d = \overbrace{\underbrace{wN^s}_{\text{labour income}} + \underbrace{\pi}_{\text{profits}}}^{Y} - T$$

↳ national income ($rK = 0$) ↳ one period

no lending

- $C + S = Y^d = wN^s + \pi - T$

but $S = 0$ $\Rightarrow$ no incentives to save
(ie there is no future)
$\hookrightarrow$ save because you can reap the benefits later!

recall $p=1$

$\therefore \; C = wN^s + \pi - T$

note: $N^s = h - \ell$

$\therefore \; C = w(h-\ell) + \pi - T$

price of $C=1$ $\rightsquigarrow$ $\underbrace{C}_{\text{spending on consumption goods}} + \underbrace{\overset{\text{price of leisure}}{w}\ell}_{\text{spending on leisure}} = \underbrace{wh}_{\text{"potential" labour income}} + \underbrace{\pi - T}_{\text{non labour income}}$

ie: the cost of leisure is w because by ↑ leisure by one unit, labour supply is reduced by one unit $\therefore$ labour income drops by w

opportunity cost

- $\therefore$ consumer's problem is:

$$\max_{(c,\ell)} \; u(c,\ell)$$
$$\text{s.t.} \; \underbrace{c + w\ell = wh + \pi - T}_{\text{Budget constraint}}$$

Solution: same as with X and Z

→ choose a reference good: consumption $(P=1)$ $(Z=C)$

∴ value of leisure in terms of consumption

is $MRS_{\ell c} = \dfrac{MU_\ell}{MU_c}$

↳ measures how many units of the consumption goods this consumer is <u>willing</u> to give up to enjoy one more unit of leisure

the cost of leisure? how many units of the consumption goods does this consumer <u>have</u> to give up to get one more unit of leisure

$(\uparrow \ell \rightarrow \downarrow N^S \rightarrow \downarrow wN^S$ by w

$\therefore \downarrow C$ by $w)$

$$\therefore \boxed{\begin{aligned} MRS_{\ell c} &= w \\ C + w\ell &= wh + \pi - T \end{aligned}}$$

$X = \ell$ $\quad P_X = w$
$Z = C$ $\quad P_Z = 1$) walras law

"=" because $MU_c > 0$
$MU_\ell > 0$

graphically: (c against ℓ)

$$c + w\ell = wh + \pi - T$$

$$c = wh + \pi - T - w\ell \qquad \text{note: } \frac{\partial c}{\partial \ell} = -w = \text{slope}$$

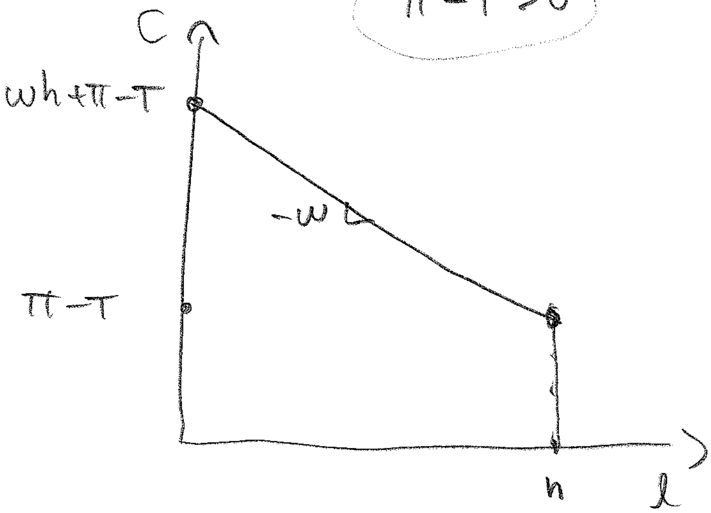
$$\ell = 0 \Rightarrow c = wh + \pi - T$$

$$\ell = h \Rightarrow c = wh + \pi - T - wh = \pi - T$$

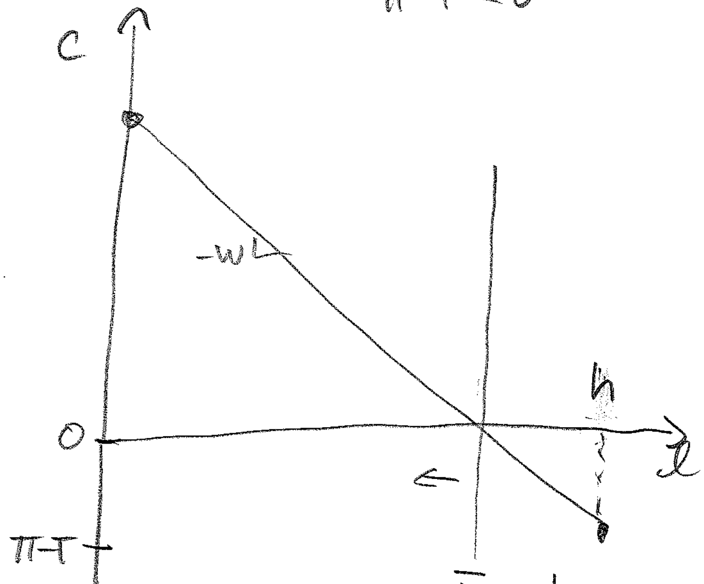
two cases: 1º $\pi - T > 0$

2º $\pi - T < 0$

$\pi - T > 0$

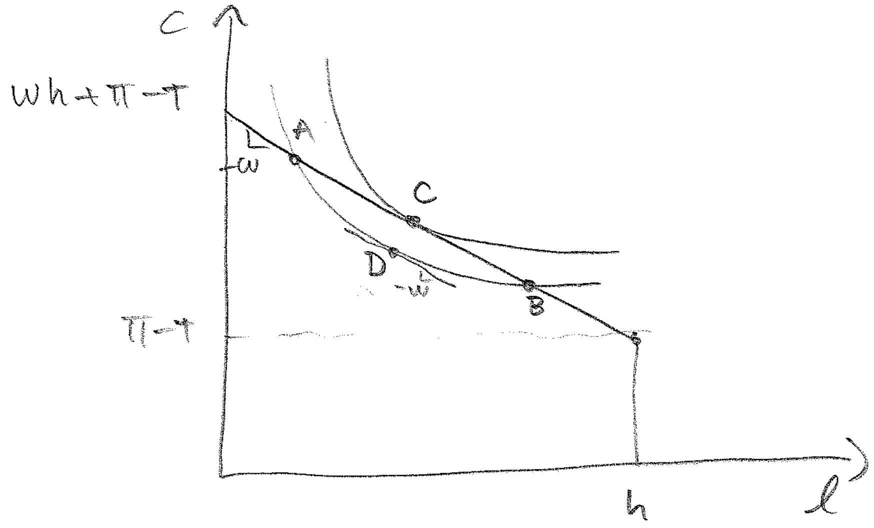


$\pi - T < 0$



$\bar{h} < h$ (otherwise $c < 0$)

focus on this case since it is more general



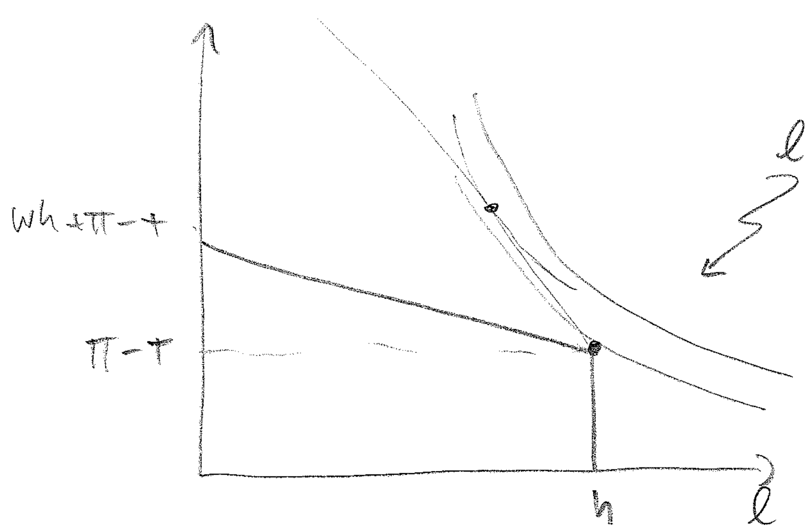
A: not optimal: $MRS_{\ell c} > w \to \uparrow \ell, \downarrow c$

B: not optimal! $MRS_{\ell c} < w \to \downarrow \ell, \uparrow c$

C: optimal! $MRS_{\ell c} = w \to$ no incentives to $\uparrow c$ or $\uparrow \ell$

D: not optimal! $MRS_{\ell c} = w$ but $c + w\ell < wh + \pi - T$
but can $\uparrow$ utility because $MU_c > 0$ and $MU_\ell > 0$

ex: implications of the "kink"



$\ell = h \to N^s = 0$

↳ <u>decides</u> not to work because wage is too low (would work for a higher wage)

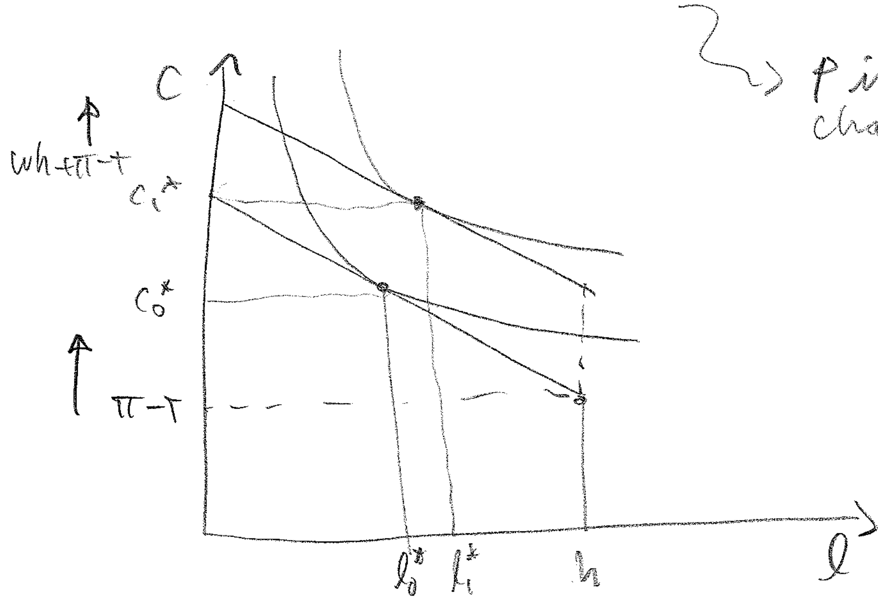
eg: called "voluntary" unemployment

↳ Here everybody who wants to work finds a job.

Comparative statics

↳ analysis of consumer's behaviour

- say non-labour income $\uparrow$ (ie: $\uparrow \pi - T$)
  ($\uparrow \pi$ or $\downarrow$ in T)



⤳ $\uparrow$ in income without changing the price of leisure
($\uparrow$ in $\underset{\uparrow}{w}N^S \longrightarrow \uparrow Y$ but also $\uparrow$ the cost leisure)

is called a "pure" income effect.

$(c_0^*, l_0^*) \rightarrow (c_1^*, l_1^*)$ ie: $\underbrace{\uparrow c \text{ and } \uparrow l}$

↓

this result comes from assumption #3 (normal goods)

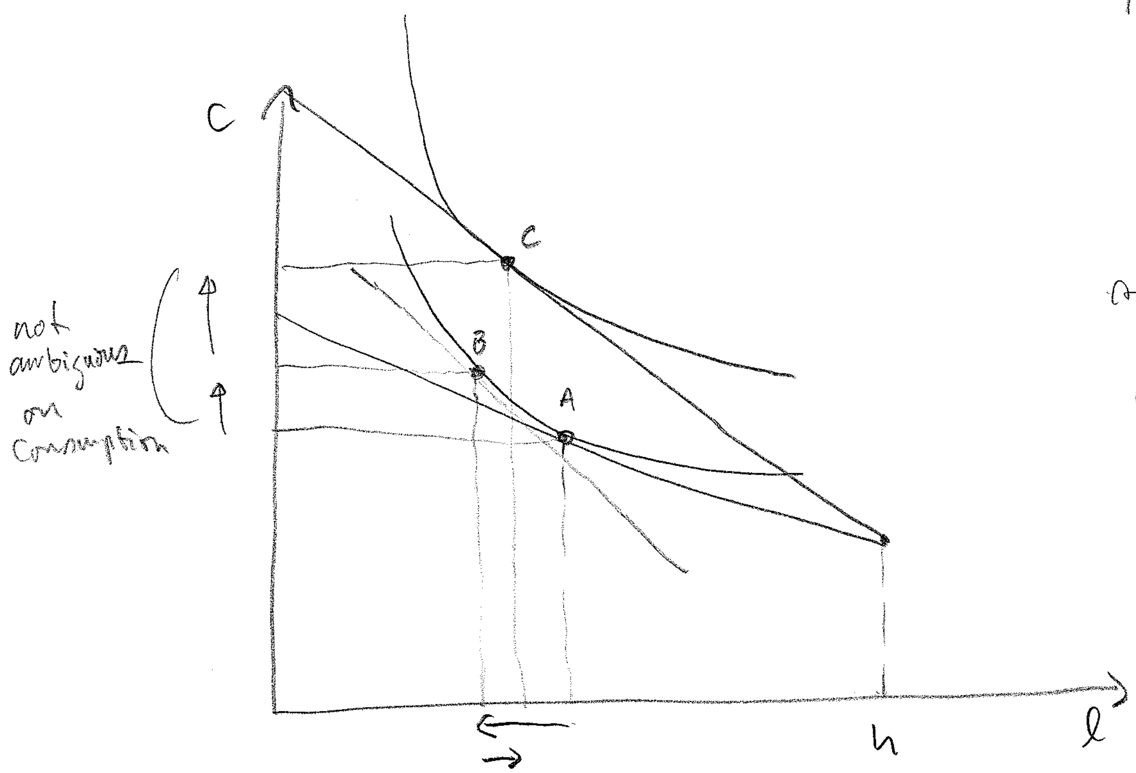
• pay labour income ↑ (because w ↑)

↳ this will have an <u>income effect</u> and a <u>substitution effect</u>

$\uparrow w \to \uparrow Y \to \uparrow \ell$ (normal good)

$\uparrow w \to \downarrow \ell$

opposite effects



substitute away from leisure (more costly)

↳ $\uparrow N^s \to \uparrow Y \to \uparrow C$

A → B : substitution effect (price of leisure ↑ ↳ ↓ ℓ)

B → C : income effect (more income → do not need to work as hard to attain same income

↳ net effect is ambiguous (on ℓ) : • if the income effect is strong, $\uparrow w \to \uparrow c \uparrow \ell$

• if the substitution effect is strong: $\uparrow w \to \downarrow \ell$ (but still ↑c)

↳ <u>Book</u> assumes that substitution effect always dominates the income effect

Example:

Say: • $u(c,\ell) = c^{1/2}\ell^{1/2}$

• $h = 6$

• $\pi - T = 8$

$\therefore$ problem $\Rightarrow$ $\max\limits_{(c,\ell)} \; c^{1/2}\ell^{1/2}$

$$\text{s.t. } \; c + w\ell \leq \underset{\substack{\uparrow \\ h}}{6}w + \underbrace{8}_{\pi - T}$$

Solution: $MRS_{\ell c} = w$

$$\boxed{\frac{c^*}{\ell^*} = w} \Rightarrow c^* = w\ell^*$$

$$\boxed{c^* + w\ell^* = 6w + 8}$$

$$\hookrightarrow w\ell^* + w\ell^* = 6w + 8$$

$$\boxed{\ell^* = 3 + \frac{4}{w}}$$

optimal solution $\leftarrow$

$$\boxed{c^* = 3w + 4}$$

1%/ $N^S(w, \pi - T) = h - \ell^*$

$$= 6 - \left[3 + \frac{4}{w}\right]$$

$$\boxed{N^S(\cdot) = 3 - \frac{4}{w}} \leftarrow \text{labour supply function}$$

$\uparrow w \rightarrow \uparrow N^S$

substitution effect

often, graph <u>inverse</u> labour supply function

↳ $P = F(Q)$

rather than $Q = F(P)$

ie: $N^S = 3 - \frac{4}{w}$

$\therefore \frac{4}{w} = 3 - N^S$

$(3 - N^S)\, w = 4$

$$\boxed{w = \frac{4}{3 - N^S}}$$

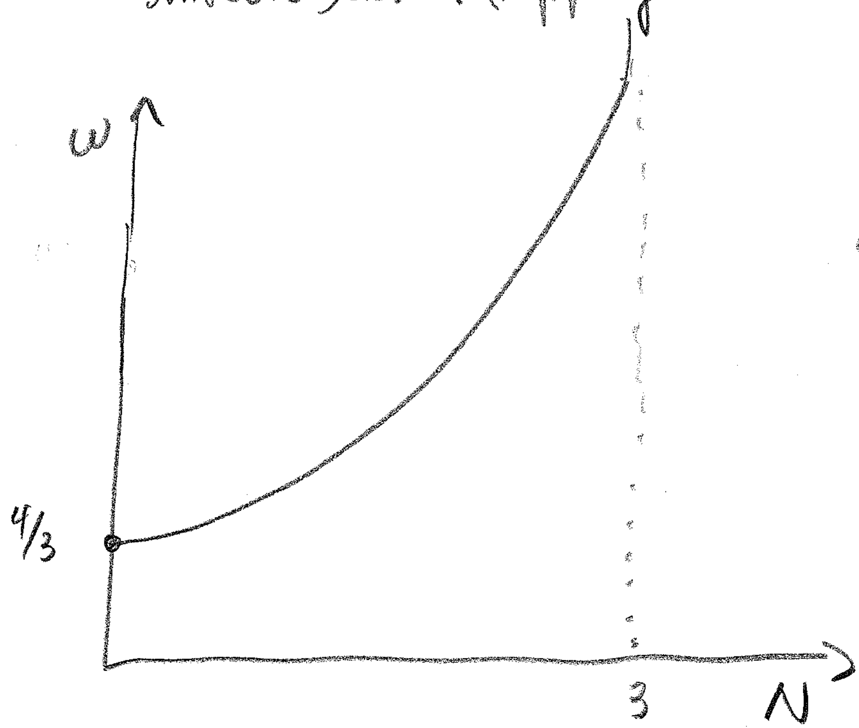
inverse labour supply

→ $\partial w / \partial N = +$

→ $N^S = 0 \Rightarrow w = \frac{4}{3}$

→ $N^S \to 3 \Rightarrow w \to \infty$

graph



substitution and income effect

Say $w = 4$

then $\ell^* = 3 + \frac{4}{4} = 4$

$c^* = 3 \cdot 4 + 4 = 16$

$\Rightarrow \boxed{\begin{array}{l} c^* = 16 \\ \ell^* = 4 \end{array}}$

$\hookrightarrow N^S = 6 - 4 = 2$

- Suppose that $w \uparrow$ to $w = 9$

then $\ell^* = 3 + \frac{4}{9} = 3\,4/9$

$c^* = 3 \cdot 9 + 4 = 31$

$\Rightarrow \boxed{\begin{array}{l} c^* = 31 \\ \ell^* = 3\,4/9 \end{array}}$

$\hookrightarrow N^S = 6 - 3\,4/9 = 2\,5/9$

$\therefore \underbrace{\uparrow w \rightarrow \uparrow N^S}$

$\hookrightarrow$ we know then that the substitution effect dominates the income effect (stronger)

- Can decompose the change from $\ell^* = 4$ to $\ell^* = 3\,4/9$ into the substitution effect and income effect

substitution effect: look at what ℓ^* would be with $w = 9$ (new wage) when utility **does not** change

recall: $u = c^{1/2} \ell^{1/2}$ $\quad MRS_{\ell c} = w \Rightarrow c = w\ell$

$u = (w\ell)^{1/2} \ell^{1/2}$

$u = w^{1/2} \ell \Rightarrow \boxed{\ell = \frac{u}{w^{1/2}}}$

$\hookrightarrow$ all possible ℓ given u and w

when $\omega = 4$, $u = 4^{1/2} \cdot 4$

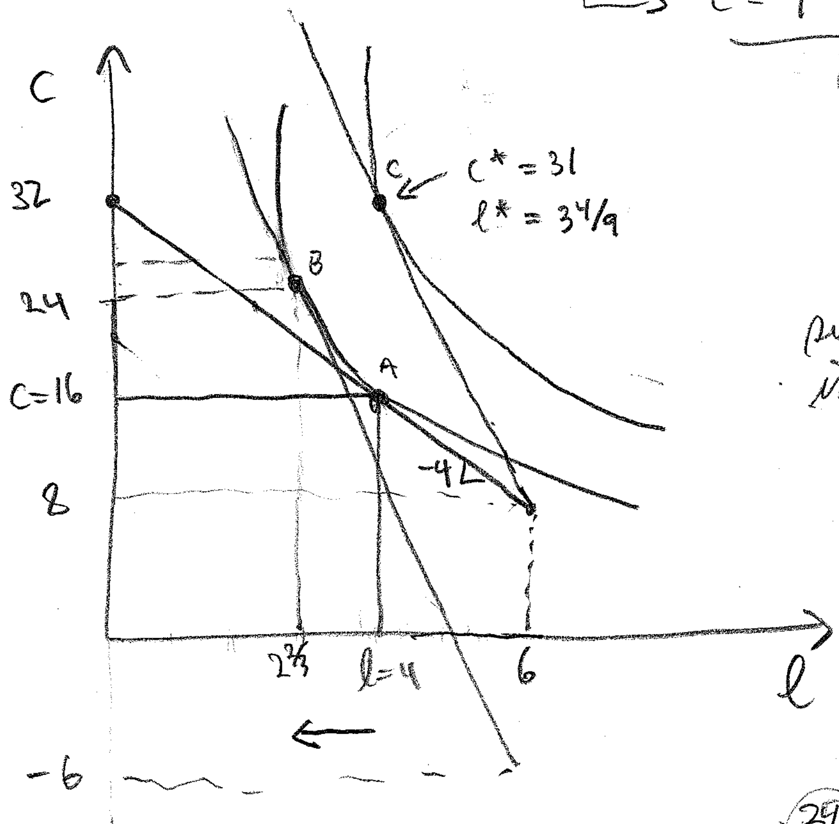
$\boxed{u = 8}$

$\therefore \quad l = \dfrac{8}{\omega^{1/2}} = \dfrac{8}{9^{1/2}} = 2\,^2/_3$

$\hookrightarrow$ if $\omega \uparrow$ from 4 to 9, l must go from 4 to $2\,^2/_3$ if utility is kept at $u=8$) $l \downarrow$ by $1\,^1/_3$

$\hookrightarrow \quad c = 9 \cdot \frac{8}{3} = 24$

$\hookrightarrow \boxed{c \uparrow \text{ by } 8}$



substitution: A to B
income: B to C

$^{24}/_9$ to $^{31}/_9$ $\to \Delta\, ^7/_9$

income effect: l goes from $2\,^2/_3$ to $3\,^4/_9$

ie $l \uparrow$ by $^7/_9 < 1\,^1/_3 = \frac{12}{9}$

$\boxed{c \uparrow \text{ by } 7}$