Example: A simple model

recall that a model is made up of 8 assumptions

1°/ Economic agents:
- N identical consumers (same preferences)
- live for 1 period
- no firms

2°/ Types of goods:
- 2 goods → x, → z that are consumed (ie: consumption goods)

3°/ Preferences:
- $u(x,z) = x^{1/2} z^{1/2}$ ie: satisfy assumption #1, #2, #3

4°/ Technology:
- $\frac{N}{2}$ consumers produce x: one unit of labour yields $\frac{2\bar{X}}{N}$
- $\frac{N}{2}$ consumers produce z: one unit of labour yields $\frac{2\bar{Z}}{N}$

5°/ Resources constraint:
- Each consumer has one unit of labour
  - ↳ since the consumers do not value leisure (ie 3°/) then each supply one unit of labour inelastically
    - ie: each consumer produces either $\frac{2\bar{X}}{N}$ of x or $\frac{2\bar{Z}}{N}$ of z ⇒ total supply $\frac{N}{2} \times \frac{2\bar{X}}{N} = \bar{X}$ same for z

6°/ Goal of Agents:
- Each consumer maximizes utility by choosing x and z

7°/ Markets [equilibrium concept]

↳ goods are exchanged in <u>competitive</u> markets

consumers/suppliers are price-takers (assume that price remain unchanged with their decisions)

- market price of x : P_x
- market price of z : P_z

8°/ no government

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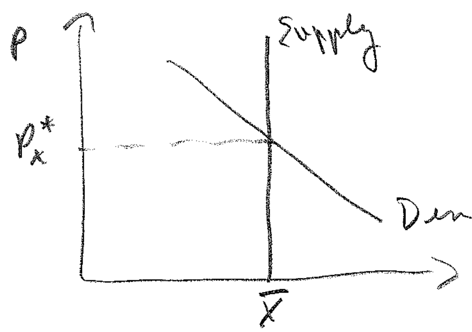
note: · finding the equilibrium of this economy requires that we find P_x and P_z

↳ given those, we can determine quantity consumed since consumers are price-takers

- Assumptions 3, 4, and 5 ⇒ · Fixed supply of x and z
  - Equ. is determined by demand

ie:



Use this example to illustrate 3 concepts:

→ Aggregation

→ Walras Law → equilibrium with respect to a reference good

→ Equilibrium

Solution:

1st/

$$\max \quad x^{1/2} z^{1/2}$$

$$\text{s.t.} \quad \underbrace{p_x x + p_z z}_{\text{spending}} = \underbrace{y_i}_{\text{income}}$$

← identical consumers except for income → $i = x, n z$

→ solve given y

solving this problem given p_x and p_z (and y) will give us the demand for x and for z

solution: $MRS_{xz} = \dfrac{p_x}{p_z}$

$$p_x x + p_z z = y$$

$$MRS_{xz} = \frac{\frac{\partial u(\cdot)}{\partial x}}{\frac{\partial u(\cdot)}{\partial z}} = \frac{\frac{1}{2} x^{-1/2} z^{1/2}}{\frac{1}{2} x^{1/2} z^{-1/2}} = \frac{z}{x}$$

$$\therefore \quad \boxed{\frac{z}{x} = \frac{p_x}{p_z}} \quad \Rightarrow \quad \boxed{z^* = \frac{p_x}{p_z} x}$$

substitute this into the budget constraint

$$p_x x^* + p_z z^* = y$$

i.e.: $p_x x^* + p_z \left(\dfrac{p_x}{p_z} x^* \right) = y$

$$2 p_x x^* = y$$

$$\boxed{x^* = \frac{y}{2 p_x}} \quad \Rightarrow \quad x^* = F(\underset{\ominus}{p_x}, \underset{\oplus}{y})$$

↑ normal good

↑ individual demand for x

→ all consumers with the same y take the same decisions

$\boxed{\text{Aggregation}}$ → from individual to aggregate demand

• say income of those producing good i is y_i

$$\therefore \quad \underbrace{\frac{N}{2}}_{\#\text{people}} \cdot \underbrace{\frac{y_x}{2P_x}}_{\substack{\text{individual}\\ \text{demand}}} + \frac{N}{2}\frac{y_2}{2P_2} = \underbrace{X^*(P_x, y_x, y_z)}_{\text{aggregate demand}}$$

$$\frac{1}{2P_x}\Big[\underbrace{\frac{N}{2}y_x + \frac{N}{2}y_2}\Big] = x^*(P_x, y_x, y_z)$$

note: • income approach = national income = Y

• product approach ⇒ Y = GDP

• GDP = value of all goods produced

i.e. $GDP = P_x \underbrace{\frac{N}{2}\left(\frac{2\bar{x}}{N}\right)}_{\text{production}} + P_z \frac{N}{2}\left(\frac{2\bar{z}}{N}\right)$

$$\boxed{GDP = p_x \bar{x} + p_z \bar{z}}$$

This type of result will always be true later on [identical consumers] →

$$\therefore \quad \boxed{X^*(P_x, \underset{\uparrow}{\underline{\underline{Y}}}) = \frac{1}{2P_x}\left[P_x\bar{x} + P_z\bar{z}\right]}$$ aggregate demand for x

national income

↳ with identical pref. all that matters is Y not y_i

$\boxed{\text{equilibrium!}}$

↳ ie find P_x and P_z

$$\underbrace{X^*(P_x, Y)}_{\text{demand}} = \underbrace{\bar{X}}_{\text{supply}} \quad \left(\text{i.e. } \frac{N}{2}\left(\frac{2\bar{x}}{N}\right) = \bar{x}\right) \Rightarrow \boxed{\text{find } P_x}$$

$$\frac{1}{2P_x}\left[P_x\bar{x} + P_z\bar{z}\right] = \bar{x}$$

$$P_x\bar{x} + P_z\bar{z} = 2P_x\bar{x}$$

$$P_z\bar{z} = P_x\bar{x} \Rightarrow \boxed{P_x^* = P_z \cdot \frac{\bar{z}}{\bar{x}}}$$

Walras law

↳ Consider an economy with n markets:

- if $n-1$ markets are in equilibrium (ie: supply = demand in each of those markets) then the remaining market is automatically in equilibrium (regardless of the price in this market)

Here, $n=2$. This means that whatever value we <u>choose</u> for P_z, the market for z is in equilibrium if $P_x = P_x^*$

ie: we know that $z^* = \frac{P_x}{P_z} x^*$ at the individual level

↳ also true in the aggregate

$$\boxed{Z^*(P_z, Y) = \frac{P_x}{P_z} X^*(P_x, Y)}$$

- we will show that $Z^*(P_z, Y) = \bar{Z}$ for any P_z when $P_x = P_x^*$
- when $P_x = P_x^*$, $X^*(P_x, Y) = \bar{X}$

$$\Rightarrow Z^*(P_z, Y) = \frac{\frac{P_z \bar{Z}}{\bar{X}} \cdot \bar{X}}{P_z}$$

$\frac{P_z \bar{Z}}{\bar{X}} \rightarrow P_x^*$ $\qquad \bar{X} \rightarrow X^*(P_x, Y)$

$$= \bar{Z}$$

$\therefore$ supply = demand for any P_z

Essentially means that we can only determine relative price $\frac{P_x}{P_z}$ not P_x and P_z individually note:

⇒: need to express everything in the economy with respect to a reference good (a numéraire) to which we can assign price 1

i.e! $p_z = 1$ → "paper money?!"
↳ need a market for money

when $p_z=1$, $p_x^* = \frac{\bar{z}}{\bar{x}}$ ⇒ $x^* = \bar{x}$, $z^* = \bar{z}$

if $p_z = 3$, $p_x^* = 3\frac{\bar{z}}{\bar{x}}$ ⇒ $x^* = \bar{x}$, $z^* = \bar{z}$

↳ p_z does not matters, all that matters is that $\frac{p_x}{p_z} = \frac{\bar{z}}{\bar{x}}$

→ note that $\frac{p_x}{p_z} = \frac{\bar{z}}{\bar{x}}$ reflects relative "scarcity"

↳ if $\bar{x} > \bar{z}$, then $\bar{z}$ is scarce relative to $\bar{z}$. Hence $p_x < p_z$