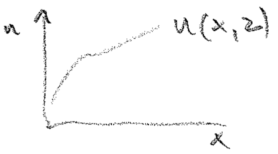
Micro foundations → develop macro model from aggregating individual behaviour

max utility
s.t. constraint

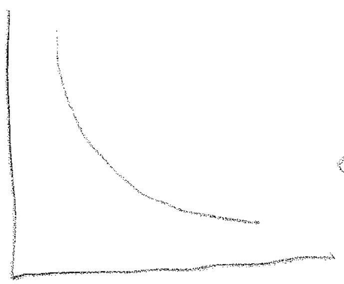
utility function: • puts a value on a consumption bundle



↳ say a consumer can consume X and Z

ie: a bundle (X,Z) has a "value" or "utility" $u = u(X,Z)$

• this function can be used to rank bundles → describe preferences



ie: • (x_1,z_1) is preferred to (x_0,z_0)

if $u(x_1,z_1) > u(x_0,z_0)$

• a consumer is indifferent between (x_1,z_1) and (x_0,z_0)

if $u(x_0,z_0) = u(x_1,z_1)$

properties of utility functions:

1st "more is preferred to less"

ie: $u(x_1,z_0) > u(x_0,z_0)$

if $x_1 > x_0$ (and z is the same)

$u(x_0,z_1) > u(x_0,z_0)$

if $z_1 > z_0$

↳ this is most accurately known as <u>positive marginal utility</u>

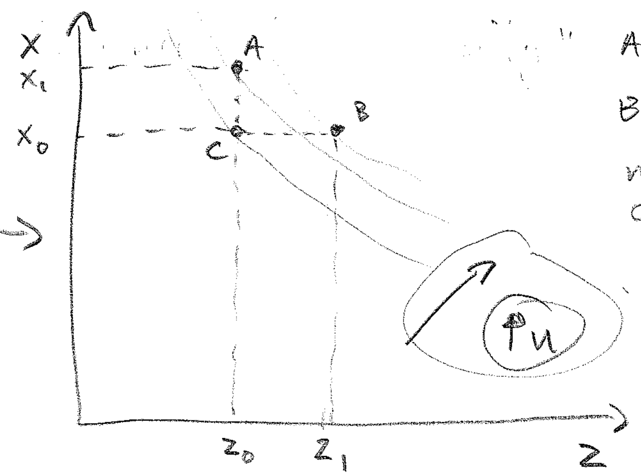
ie: this property holds if $\frac{\partial u(x,z)}{\partial x} > 0$

$\frac{\partial u(x,z)}{\partial z} > 0$

measures ↑ in welfare following ~~an~~ Δ in x ← $\frac{\partial u}{\partial x} = MU_x$

it means that ↑x always ↑u and that ↑z always ↑u [non-satiation → always want more]

explain indifference curves



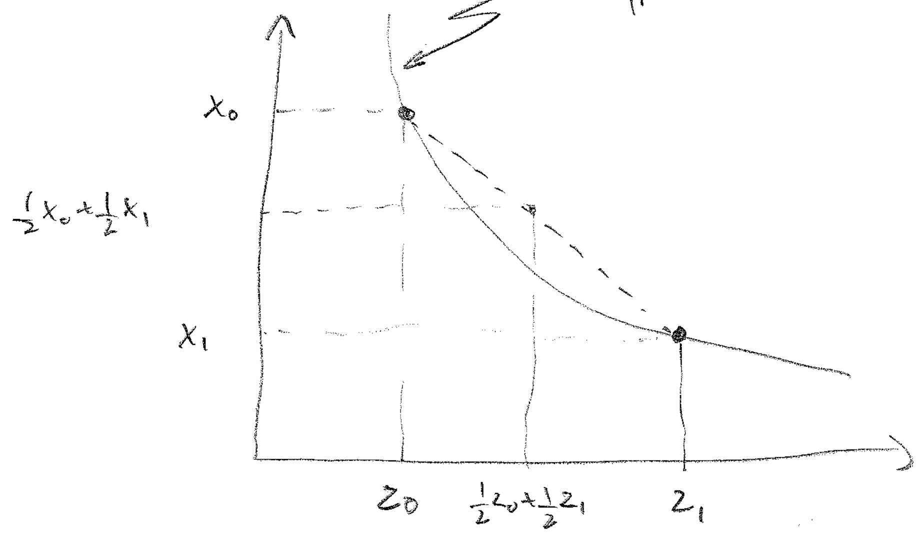
A is prefered to C
B is prefered to C
not enough info to compare A and B

→ all it says

2nd "preference for diversity" (decreasing marginal utility)

say $u(x_0, z_0) = u(x_1, z_1)$

indifference must look like that



any bundle which is a combination of these two extreme bundles are prefered to them

$$u\left(\tfrac{1}{2}x_0 + \tfrac{1}{2}x_1,\ \tfrac{1}{2}z_0 + \tfrac{1}{2}z_1\right) > \underbrace{u(x_0, z_0) \text{ or } u(x_1, z_1)}$$

also means that any bundles on the indifference curve are also dominated

$\frac{\partial MU_x}{\partial x} < 0$ ↑x → ↓MU_x

ie: more is better but less and less satisfying | getting fed up!

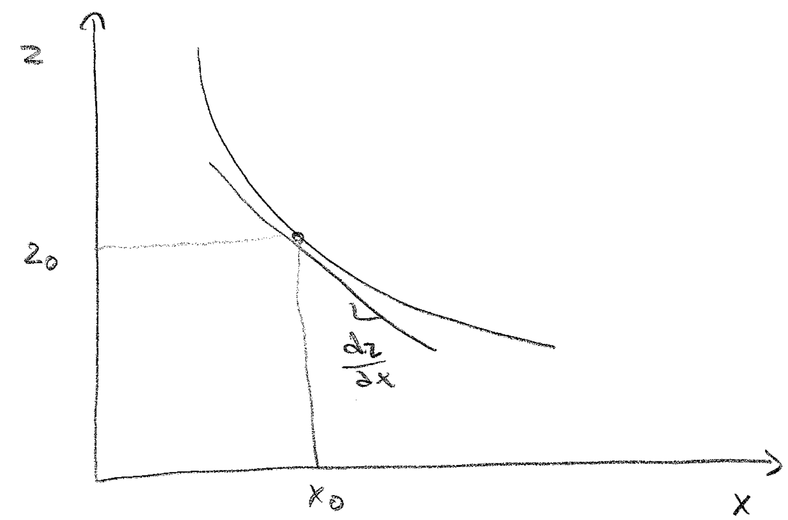
3°/ x and z are normal goods (ie not inferior good)

↳ ie: ↑ income ⟶ ↑ in the consumption of x and that of z at the same time

eg: fast food : inferior good
expensive restaurant : normal good
potato : inferior good

Marginal rate of substitution (MRS) → measures willingness to pay

↳ measures the value of a good, not in terms of utility, but in terms of another good.



→ suppose that a consumer consumes (x_0, z_0): How many units of z would this consumer be willing to give up to get an extra unit of x?

↳ ie: the value of one unit of x in terms of good z

- to answer this question, we need to figure out the value of $\frac{dz}{dx}$ which <u>keep utility constant</u>

↳ if the trade ↑ utility for example, it means that the consumer would be willing to give up even more units than $\frac{dz}{dx}$

we know that: $u = U(x, z)$ therefore

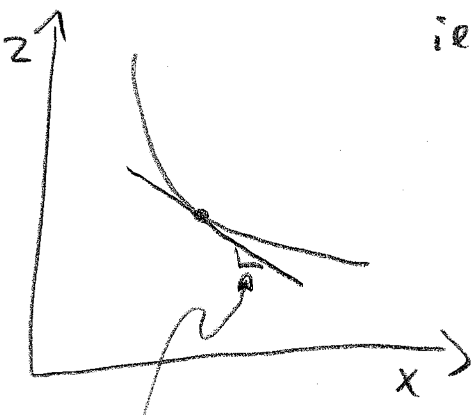
$$du = \frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial z} \cdot dz$$

∴ set $du = 0$ → no change in utility and calculate $\frac{dz}{dx}$

ie: $$0 = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial z} dz$$

$$\therefore \frac{dz}{dx} = - \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial z}} = - \frac{MU_x}{MU_z}$$

↳ call that the MRS_{xz}



$= \frac{dz}{dx}$!

ie the slope of an indifference curve is $- MRS_{xz}$

ex: $MRS_{xz} = 3$

means that a consumer ~~is willing~~ to give up 3 units of z to get one unit of x

↳ sort of a "relative price" assigned by the consumer to a good

This <u>willingness</u> to pay must be compared to the <u>capacity</u> to pay

↳ this is given by the budget/resources constraint

- say a consumer has income y.
- the price of good x is p_x and that of good z is p_z

$$\therefore \quad \underbrace{p_x x + p_z z}_{\text{expenditures}} = \underbrace{y}_{\text{income}}$$

note:
$p_x x + p_z z < y$ is not possible because $MU_x > 0$ and $MU_z > 0$

ie: it is possible to ↑U by ↑x and ↑z at the same time while keeping the budget constraint satisfied

can look at $\frac{dz}{dx}$ based on this budget constraint

ie: $dy = p_x\,dx + p_z\,dz$

$$dy = 0 \Rightarrow \frac{dz}{dx} = -\boxed{\frac{p_x}{p_z}}$$

↳ this tells us how many units of z one <u>has</u> to give up to get an extra unit of x (according to the market because p_x and p_z are fixed on the market)

ie: $\frac{p_x}{p_z}$ is the <u>market</u> value of x

MRS_{xz} is the consumer's own valuation of x

they are not necessarily equal to each other utility maximization $\Rightarrow$ best choice of x and z given the constraint

Say $\underbrace{MRS_{XZ}}$ $>$ $\underbrace{\frac{P_X}{P_Z}}$

consumer values x more than the market

ex: $MRS_{XZ} = 3$, $\frac{P_X}{P_Z} = 1$ $\rightarrow$ the consumer is willing to give up 3 units of z to get one more unit of x but on the market, he only has to give up 1 unit of z

ie: buying "x" at a bargain price!

$\therefore$ $MRS_{XZ} > \frac{P_X}{P_Z}$ $\rightarrow$ buy more ~~x~~ and reduce spending on z

ie: $\uparrow x$ $\downarrow z$

but, recall that $\uparrow x \rightarrow \downarrow MU_X$

$\downarrow z \rightarrow \uparrow MU_Z$

and that $MRS_{XZ} = \frac{MU_X \downarrow}{MU_Z \uparrow}$ $\therefore MRS_{XZ} \downarrow$

ie: as this consumer buys more x, its valuation of x goes down

$\Rightarrow$ continue buying x until $\underbrace{MRS_{XZ}}_{\text{valuation}} = \underbrace{\frac{P_X}{P_Z}}_{\text{market value}}$

Say $MRS_{xz} < \frac{P_x}{P_z}$

$\hookrightarrow$ consumer values x less than the market price
[ie: at the current allocation, the consumption of x is too high]

$\therefore \downarrow x \ \uparrow z \Rightarrow$ $\begin{array}{l} \downarrow x \to \uparrow MU_x \\ \uparrow z \to \downarrow MU_z \end{array}$ $\Rightarrow \uparrow MRS_{xz}$
until
$MRS_{xz} = \frac{P_x}{P_z}$

$\therefore$ the optimal bundle is found as the one for which $\boxed{MRS_{xz} = \frac{P_x}{P_z}}$

ie: for this allocation, the consumer's valuation of x coincides with the market value.

graphically:

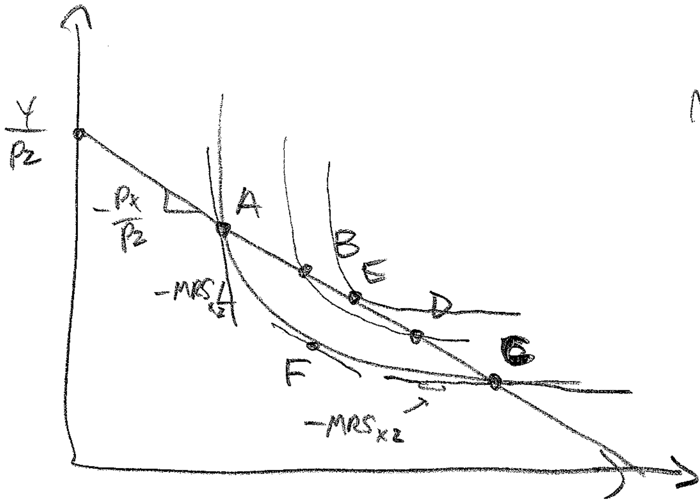
→ already know how to graph indifference curves

→ the budget constraint: get an expression for z as a function of x

$$p_x x + p_z z = y$$
$$p_z z = y - p_x x$$
$$\boxed{z = \frac{y}{p_z} - \frac{p_x}{p_z} x}$$

$\hookrightarrow$ this equation gives all the possible values of z given x

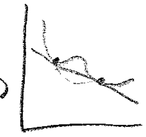
note: $\frac{dz}{dx} = -\frac{p_x}{p_z}$



reasonable too

Assumption #1
↳ rules out allocation F

Assumption #2
↳ E is the unique solution to the max problem



for the allocation A, $MRS_{xz} > \frac{p_x}{p_z} \longrightarrow \uparrow x \downarrow z$

∴ move to B
B is preferred to A (higher utility)

for allocation C, $MRS_{xz} < \frac{p_x}{p_z} \longrightarrow \downarrow x \uparrow z$

∴ move to D
D is preferred to C (higher utility)

allocation E is optimal $\Rightarrow \boxed{MRS_{xz} = \frac{p_x}{p_z}}$

<u>But</u> what about allocation F?

at F, $MRS_{xz} = \frac{p_x}{p_z}$ too!

is this allocation optimal too?

↳ no! at F $p_x x + p_z z < y$

↳ not all the budget is spent

$(MU_x > 0 \quad MU_z > 0)$ $\Rightarrow$ can $\uparrow u$ without having to trade x for z

can $\uparrow x$ $\uparrow z$ at the same time to $\uparrow u$

the optimal allocation must satisfy

$$\left.\begin{aligned} MRS_{xz} &= \frac{p_x}{p_z} \\ p_x x + p_z z &= y \end{aligned}\right\} \text{together}$$

example:

$$u(x,z) = x^{1/3} z^{2/3}$$

$$MRS_{xz} = \frac{MU_x}{MU_z} = \frac{\frac{1}{3} x^{-2/3} z^{2/3}}{\frac{2}{3} x^{1/3} z^{-1/3}} = \frac{1}{2}\frac{z}{x}$$

$\therefore$ optimality requires: $\frac{1}{2}\frac{z}{x} = \frac{P_x}{P_z}$

$$\therefore \boxed{z = 2\frac{P_x}{P_z}x}$$

feasibility requires $\boxed{P_x x + P_z z = y}$

together

$$P_x x + P_z\left(2\frac{P_x}{P_z}x\right) = y$$

$$P_x x + 2P_x x = y$$

$$3P_x x = y$$

$$\boxed{x = \frac{y}{3P_x}} \Rightarrow z = 2\frac{P_x}{P_z}x$$

$$= 2\frac{P_x}{P_z}\frac{y}{3P_x}$$

$$\boxed{z = \frac{2}{3}\frac{y}{P_z}}$$

ex: $\left(\begin{matrix} y=9 \\ P_x=1 \\ P_z=2 \end{matrix}\right) \Rightarrow \begin{matrix} x^*=3 \\ z^*=3 \end{matrix}$

note: • these are demand function

ie: $Q(P)$

• $\uparrow P \rightarrow \downarrow Q$

• $\uparrow y \rightarrow \uparrow Q$ (normal good)

note: used to see "inverse" demand functions

ie: $P(Q)$ here: $P_x = \frac{1}{3}\frac{y}{x}$