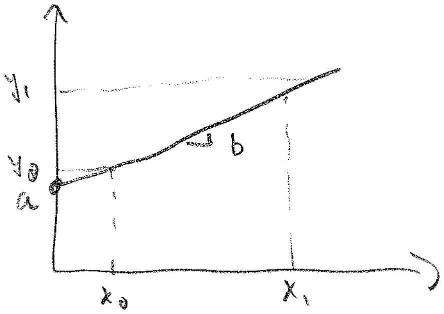
## Math Review

- simplest function: $y = a + bx$

First → then




slope ⇒ • measures how fast $y$ changes when $x$ changes → "speed" of change
• sign of the relationship ⊕ or ⊖

commonly used functions:

- $y = a + bx$
  ↳ algebra: ex $y = 3$
  $3 - a = bx$
  $x = \frac{3-a}{b}$


$x^a \cdot x^b = x^{a+b}$

$x^a \cdot x^{-b} = x^{a-b}$

$x^{-a} = \frac{1}{x^a}$

ex: $x^{1/2} x^{-1} = x^{1/2 - 1} = x^{-1/2} = \frac{1}{x^{1/2}}$

then →

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}$$

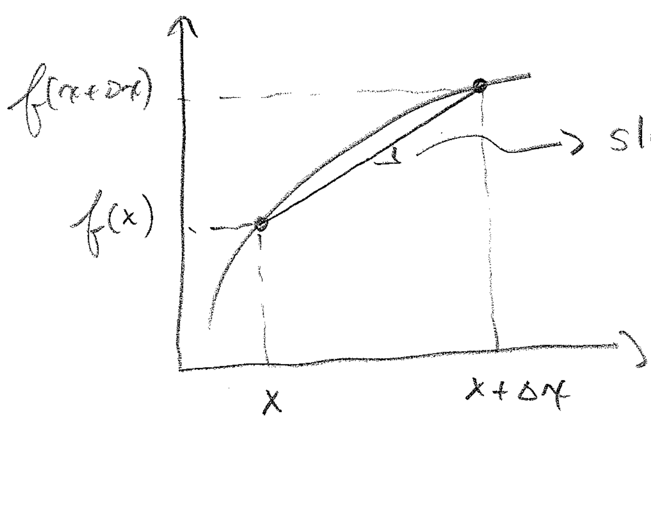
$$y_1 = a + bx_1$$

$$y_0 = a + bx_0$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{a + bx_1 - (a + bx_0)}{x_1 - x_0}$$

$$= \frac{b(\cancel{x_1 - x_0})}{\cancel{x_1 - x_0}} = b$$

- derivatives → equivalent of "slope" but for (possibly) non linear functions
  ie: $y = f(x)$

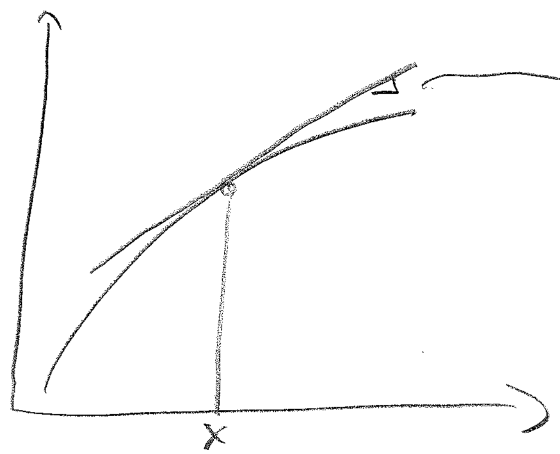



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{x + \Delta x - x}$$

$$= \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Say $\Delta x \to 0$, then $\frac{f(x+\Delta x)-f(x)}{\Delta x} \Rightarrow \frac{\partial f(x)}{\partial x} \underset{\text{or}}{=} f'(x)$

$\left( \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = f(x) \right)$




- derivative is the slope of the tangent line at $x$
- like the "slope", it measures how fast $f(\cdot)$ changes as x is varied by a small amount $\Delta x$

ex: $u = u(c)$

say $u'(c) = 3$. it means that $\uparrow c$ increases "u" by 3 [at the margin (tiny amount)]

rules:

$$f(x) = c \Rightarrow f'(x) = 0$$

$$f(x) = x^a \Rightarrow \frac{\partial f(x)}{\partial x} = a x^{a-1}$$

$$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$$

$$f(x) = g(x) + h(x) \Rightarrow f'(x) = g'(x) + h'(x)$$

ex: $f(x,y) = x^a y^b \Rightarrow \frac{\partial f(x,y)}{\partial x} = a x^{a-1} \underline{y^b}$

does not vary with $x$

$$\frac{\partial f(x,y)}{\partial y} = b x^a y^{b-1}$$

$f(x) = a + bx \qquad f'(x) = 0 + b = b$

## Differential

- Say $dx$ is close to zero:
  in this case, we know that

$$\frac{f(x+dx) - f(x)}{dx} = \frac{\partial f(x)}{\partial x}$$

$$\therefore \underbrace{f(x+dx) - f(x)}_{} = \frac{\partial f(x)}{\partial x} \cdot dx$$

$$\underbrace{df}_{\text{change in } f} = \underbrace{\frac{\partial f(x)}{\partial x}}_{\text{derivative}} \cdot \underbrace{dx}_{\text{change in } x}$$

- Total differential

↳ can use the same rule for $f(x,y)$ when $x$ and $y$ change at the same time

$$\underbrace{f(x+dx, y+dy) - f(x,y)}_{df} = \underbrace{\underbrace{\frac{\partial f(x,y)}{\partial x} dx}_{\substack{\text{partial change} \\ \text{due to } x}} + \underbrace{\frac{\partial f(x,y)}{\partial y} dy}_{\substack{\text{partial change} \\ \text{due to } y}}}_{\text{sum of change}}$$

ex: $f(x,y) = x + 3y$

$$\left.\begin{array}{l} \frac{\partial f(x,y)}{\partial x} = 1 \\ \frac{\partial f(x,y)}{\partial y} = 3 \end{array}\right) \Rightarrow \boxed{df = dx + 3\,dy}$$

use: setting $df = 0$ gives all the changes in $x$ and those in $y$ that keeps $f$ constant ie: $0 = dx + 3dy$

$3\,dy = -dx$

$\frac{dy}{dx} = -\frac{1}{3}$ to keep $f$ constant and increase $x$ by 1, one needs to <u>decrease</u> $y$ by $1/3$

That is, how many units of $y$ does one has to give up to get an extra unit of $x$

ex: $u = x^{1/2} y^{1/3}$ $\rightarrow du = \frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial y} dy$

$\frac{\partial u}{\partial x} = \frac{1}{2} x^{-1/2} y^{1/3}$ $\qquad du = 0 \Rightarrow \frac{dy}{dx} = -\frac{\partial u / \partial x}{\partial u / \partial y}$

$\frac{\partial u}{\partial y} = \frac{1}{3} x^{1/2} y^{-2/3}$

$\hookrightarrow \frac{dy}{dx} = -\frac{\frac{1}{2} x^{-1/2} y^{1/3}}{\frac{1}{3} x^{1/2} y^{-2/3}}$

$$\boxed{\frac{dy}{dx} = \frac{3}{2}\frac{y}{x}}$$