MAT 1332 Winter 2012 Assignment 4

Due February 29 in Class.

Late assignments will **not** be accepted; **nor** will unstapled assignments.

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Student Name _____ Student Number _____

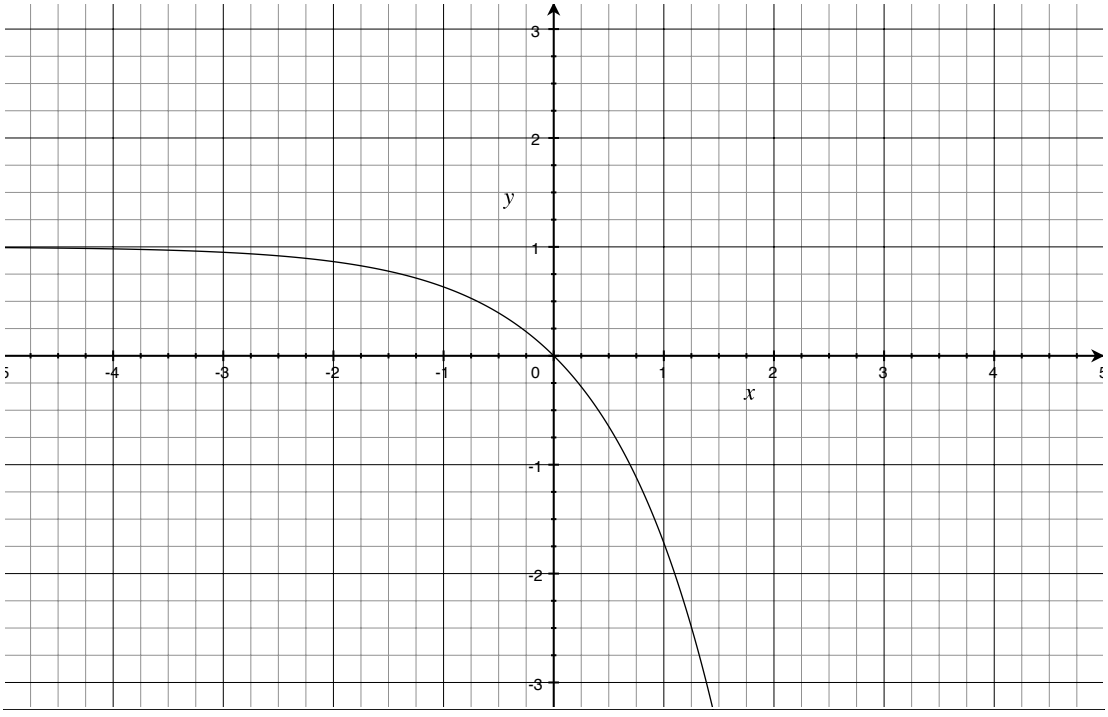
By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature _____

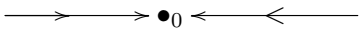
QUESTION 1. Consider the differential equation

$$\frac{dx}{dt} = 1 - e^x.$$

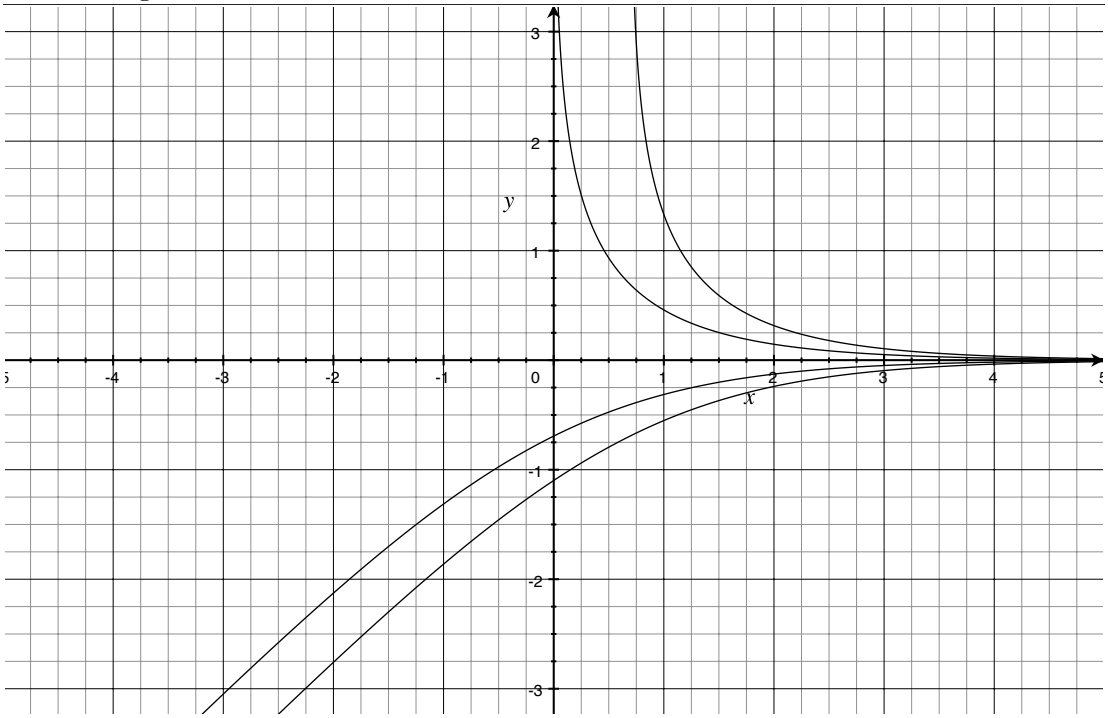
(a) Graph the rate of change as a function of the state variable over the interval $-2 \leq x \leq 2$.



(b) Draw the phase line diagram.



(c) Give a rough sketch of some of the solution curves.



QUESTION 2. Consider the complex number $z = 5 - \sqrt{2}i$.

(a) Find the complex conjugate $\overline{z}$.

$$\overline{z} = 5 + \sqrt{2}i$$

(b) Find the modulus $|z|$.

$$|z| = \sqrt{5^2 + \sqrt{2}^2} = \sqrt{25 + 2} = \sqrt{27}$$

(c) Find the inverse z^{-1}.

$$z^{-1} = \frac{1}{z} = \frac{1\overline{z}}{z\overline{z}} = \frac{5 + \sqrt{2}i}{\sqrt{27}} = \frac{5}{\sqrt{27}} + \frac{\sqrt{2}i}{\sqrt{27}}$$

QUESTION 3. Consider the autonomous differential equation

$$\frac{dx}{dt} = cx + x^2$$

with $c > 0$.

(a) Find the points of equilibrium.

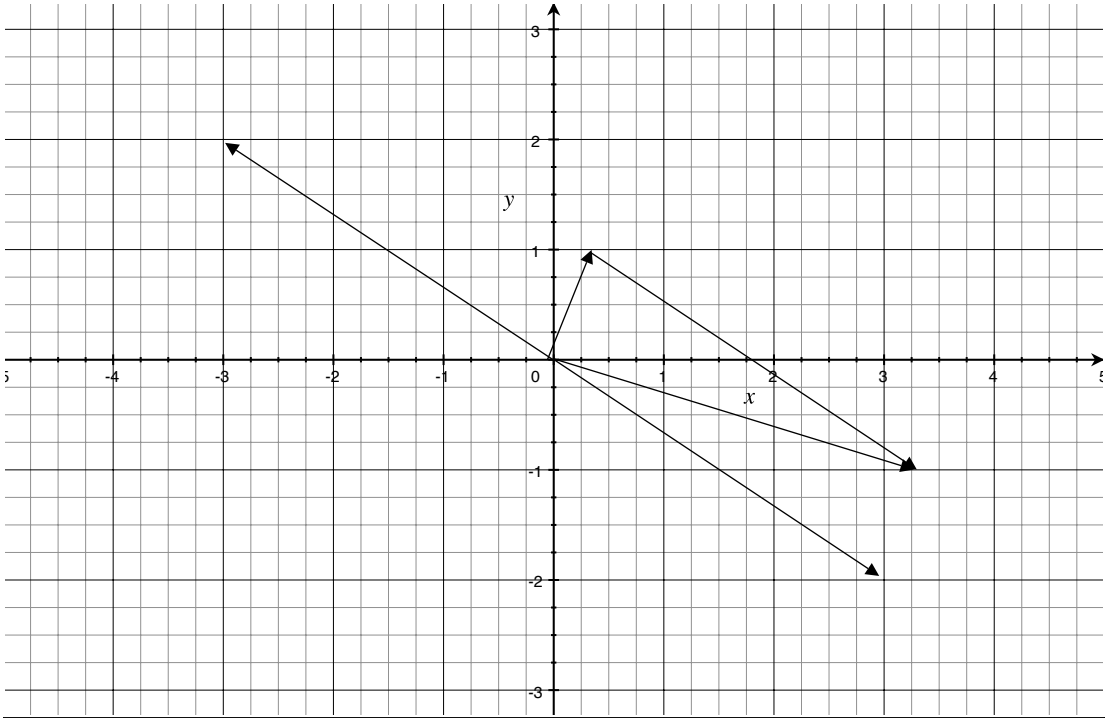
$$cx + x^2 = 0 \Rightarrow x(c+x) = 0 \Rightarrow x = 0, -c$$

(b) Find the stability of equilibrium points.

$$\left(\frac{dx}{dt}\right)'\Big|_{x=0} = c + 2(0) = c > 0 \quad \text{unstable}$$
$$\left(\frac{dx}{dt}\right)'\Big|_{x=-c} = c + 2(-c) = -c < 0 \quad \text{stable}$$

QUESTION 4. Consider the complex numbers $z = -3 + 2i$ and $w = \frac{1}{3} + i$.

(a) Plot the z, w and their difference $w - z$ in the plane.



(b) Express z and w in polar coordinates. Find the product zw.

$$r_z = |z| = \sqrt{(-3)^2 + 2^2} = \sqrt{13}, \quad \theta_z = \arctan\left(-\frac{2}{3}\right) + \pi$$
$$z = \sqrt{13}e^{i\left(\arctan\left(-\frac{2}{3}\right)+\pi\right)}$$

$$r_w = |w| = \sqrt{\left(\frac{1}{3}\right)^2 + 1^2} = \sqrt{\frac{10}{9}}, \quad \theta_w = \arctan(3)$$

$$w = \sqrt{\frac{10}{9}} e^{i(\arctan(3))}$$

$$zw = \sqrt{13}\sqrt{\frac{10}{9}} e^{i\left(\arctan\left(-\frac{2}{3}\right)+\pi+\arctan(3)\right)}$$

QUESTION 5. Express the following complex numbers in polar form (with $\theta \in [0, 2\pi)$):

(a) 2+2i;

$$\sqrt{8} e^{i\arctan(1)} = \sqrt{8} e^{i\frac{\pi}{4}}$$

(b) i;

$$e^{i\frac{\pi}{2}}$$

(c) 5;

$$5e^{i0}$$

(d) $\sqrt{3} + i$.

$$\sqrt{\sqrt{3}^2 + 1^2} e^{i\arctan(\frac{1}{\sqrt{3}})} = \sqrt{10} e^{i\arctan(\frac{1}{\sqrt{3}})}$$