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mid1 v5. pdf.

University of Ottawa
MAT 1330C Midterm Exam

October 19, 2011- Duration: 80 minutes. Instructor: Olga Vassilieva

Family Name: _____

First Name: _____

DGD 1 DGD 2 DGD 3 DGD 4

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed. No exceptions.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Please do not detach the pages.
- Good Luck!

Student number:_____, Total marks: _____ out of 30

Problem	1	2	3	4	5	6	7
Marks							

Question 1. Use the definition of derivative to calculate the derivative of

$$f(x) = 6x + 3.$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{6(x+h)+3-[6x+3]}{h} =$$

$$= \lim_{h\to 0} \frac{6x+6h+3-6x-3}{h} = \lim_{h\to 0} \frac{6h}{h} = 6$$

derivative = 6

Question 2. (a) Estimate the limit

$$\lim_{x\to 0^+} \frac{\sqrt{2-2\cos(6x)}}{x} = \lim_{x\to 0^+} f(x)$$

using a sequence. Two terms are sufficient. (Hint: make sure your calculator is set to radians!)

$X_1 = 0.1$ $\quad f(0.1) = \dfrac{\sqrt{2-2\cos(0.6)}}{0.1} \approx 5.9104$

$X_2 = 0.01$ $\quad f(0.01) = \dfrac{\sqrt{2-2\cos(0.06)}}{0.01} \approx 5.999$

Estimate for (a): $\boxed{6}$

(b) Estimate the limit

$$\lim_{x\to 0^-} \frac{\sqrt{2-2\cos(6x)}}{x}$$

using a sequence. One term is sufficient.

$X_3 = -0.01$ $\quad f(-0.01) = \dfrac{\sqrt{2-2\cos(-0.06)}}{-0.01} \approx -5.999$

Estimate for (b): $\boxed{-6}$

(c) Does the limit

$$\lim_{x\to 0} \frac{\sqrt{2-2\cos(6x)}}{x}$$

exist? $\boxed{\text{No}}$ Briefly explain your reasoning.

The left-side limit is not equal to the right-side limit. Thus, $\lim\limits_{x\to 0} \dfrac{\sqrt{2-2\cos(6x)}}{x}$ does not exist.

Question 3. (a) What is the domain of

$$f(x) = \frac{e^{5x} + x^6}{6x + 1}?$$

Domain = $\{x \in \mathbb{R},\ x \neq -\frac{1}{6}\}$

(b) Calculate the derivative of this function. (Do not simplify your result.)

$$f'(x) = \frac{(e^{5x} + x^6)' \cdot (6x+1) - (e^{5x} + x^6)(6x+1)'}{(6x+1)^2} =$$

$$= \frac{(5e^{5x} + 6x^5)(6x+1) - (e^{5x} + x^6) \cdot 6}{(6x+1)^2}$$

Question 4. Evaluate the limit

$$\lim_{x \to 6} \frac{\sin(x^2) + \cos^2(2x) + e^x}{x^2 - 3x + 5} = \lim_{x \to 6} f(x)$$

You do **not** need to simplify your answer. You may use any results from class.

limit = $\lim_{x \to 6} f(x) = f(6) = \frac{\sin(36) + \cos^2(12) + e^6}{36 - 18 + 5} \simeq 17.52$

Justify your answer in a short sentence. continuous

$f(x)$ is a ratio of two functions: $[\sin(x^2) + \cos^2(2x) + e^x)$ and $x^2 - 3x + 5$. The numerator is a sum of three continuous functions, it is a continuous function as well. The denominator is a polynomial f-n and it is not equal to zero at any point x, since $\Delta = b^2 - 4ac = 9 - 4 \cdot 1 \cdot 5 < 0$. Thus, $f(x)$ is continuous everywhere. That means that the value of the limit is obtained simply by substituting 6 for x.

Question 5. Evaluate the following limit exactly, showing your work:

$$\lim_{h\to 0} \frac{(6+h)^2 - 6^2}{\sqrt{6+h} - \sqrt{6}} =$$

$$= \lim_{h\to 0} \frac{36+12h+h^2-36}{\sqrt{6+h} - \sqrt{6}} = \lim_{h\to 0} \frac{12h+h^2}{\sqrt{6+h} - \sqrt{6}} =$$

$$= \lim_{h\to 0} \frac{(12h+h^2)(\sqrt{6+h} + \sqrt{6})}{(\sqrt{6+h} - \sqrt{6})(\sqrt{6+h} + \sqrt{6})} =$$

$$= \lim_{h\to 0} \frac{(12h+h^2)(\sqrt{6+h} + \sqrt{6})}{6+h-6} =$$

$$= \lim_{h\to 0} \frac{h(12+h)(\sqrt{6+h} + \sqrt{6})}{h} =$$

$$= \lim_{h\to 0} (12+h)(\sqrt{6+h} + \sqrt{6}) =$$

$$= 12 \cdot 2\sqrt{6} = 24\sqrt{6}$$

limit = $\boxed{24\sqrt{6}}$

Question 6. [8 points] Consider the discrete-time dynamical system (DTDS)

$$M_{t+1} = -0.6M_t + 6$$

(a) [1 point] Find the updating function of the DTDS. $f(M) = -0.6M + 6$

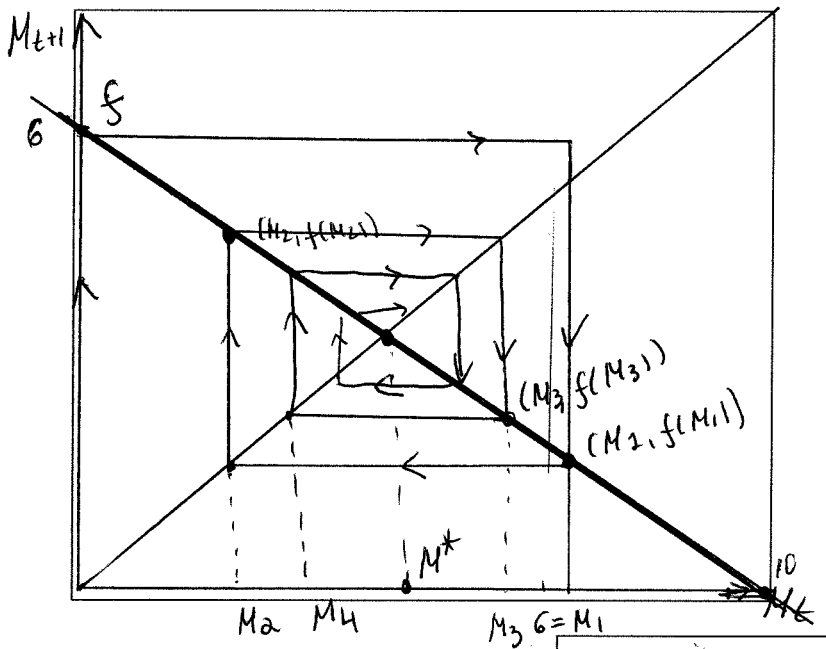
(b) [1 point] Find the equilibrium point of the DTDS. $M^* = \frac{b}{1-a} = \frac{6}{1.6} = 3.75$

(c) [2 points] Give the general solution formula for the DTDS:

$M_t =$ $(-0.6)^t M_0 + 3.75(1 - (-0.6)^t)$

(d) [1 point] Calculate M_{10} if $M_0 = 0$. $M_{10} =$ 3.7273

(e) [2 points] Graph the updating function and draw the cobweb diagram of the DTDS, starting from $M_0 = 0$ for at least 4 steps.



(f) [1 point] Is the equilibrium point stable or unstable? Stable

Question 7. (a) Find the critical point(s) of the function $f(x) = (-x^2 - 3x - 3)e^{-x}$.

$$f'(x) = (-x^2-3x-3)' \cdot e^{-x} + (-x^2-3x-3)(e^{-x})' =$$

$$= (-2x-3)e^{-x} - e^{-x}(-x^2-3x-3) =$$

$$= (-2x-3)e^{-x} + e^{-x}(x^2+3x+3) =$$

$$= e^{-x}(-2x-3+x^2+3x+3) = e^{-x}(x^2+x) = 0$$

$$f'(x) = e^{-x} \cdot x(x+1) = 0$$

$$x_1 = 0 \quad , x_2 = -1$$

critical points: $x_1 = 0$, $x_2 = -1$

(b) Find the intervals where the function is increasing and where it is decreasing.

$f'(x)$

$f'(1) > 0$

$f'(-\frac{1}{2}) < 0$

$f'(-2) > 0$

increasing: $(-\infty, -1)$ and $(0, +\infty)$ decreasing: $(-1, 0)$.