

Preparation for final

I really want all of you to do as well as possible in your final so in order for you to do that I will need you **P.R.E.P.A.R.E.**

P **Plan your work.**

Know what you want to do. Your final goal at the end of this course can range from getting an A+, to doing your best, to passing. Everyone can't have an A so I just want all of you to try your best. Just remember that you can't do that unless you put some work in.

R **Read all of your material.**

Osmosis does not work with Math. You have to read it to get it. You have a text and class notes (I hope) so make sure that you use them well.

E **Explain your Steps**

With every problem you do, you need to let the professor know that you really know the concepts behind the things that you are doing. IF you can do that, then if you do any calculation mistakes he or she is more likely to give you part marks for your efforts if it is clear that you were on the right track.

P **Practice, Practice, Practise!!!**

The problems won't do themselves; you have to do them so get used to it before the final.

A **Avoid using the solution manual until after you have attempted the problems**

You will not have solutions to the problems during the exam. If you can only do problems now while you are looking at the solution manual then you will have a problem later when you do not have access to it. Use it only to check that you have done the correct steps and or have the correct answer. Remember that sometimes there is more than one way to do a problem so you do not always have to have the same steps as the manual. You just need to have the same final answer.

R **Review often.**

You would be surprised how many little things you can forget in a short period of time so it will be important for you to review your material as much as you can.

E **Enjoy your work.**

Calculus is actually fun. Some of you might just not have noticed it yet. Remember that.

Trigonometric Functions

1. $\csc \theta = \frac{1}{\sin \theta}$
2. $\sec \theta = \frac{1}{\cos \theta}$
3. $\tan \theta = \frac{\sin \theta}{\cos \theta}$
4. $\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$
5. $\sin^2 \theta + \cos^2 \theta = 1$
6. $\sec^2 \theta = 1 + \tan^2 \theta$
7. $\csc^2 \theta = 1 + \cot^2 \theta$
8. $\sin(-\theta) = -\sin \theta$
9. $\cos(-\theta) = \cos \theta$
10. $\sin(\theta + 2\pi) = \sin \theta$
11. $\cos(\theta + 2\pi) = \cos \theta$
12. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
13. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
14. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
15. $\sin 2x = 2 \sin x \cos x$
16. $\cos 2x = \cos^2 x - \sin^2 x$

Product & Quotient Rules

1. Product Rule: $(fg)' = f'g + g'f$
2. Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

Log Rules

1. $\log AB = \log A + \log B$
2. $\log \frac{A}{B} = \log A - \log B$
3. $\log A^p = p \log A$
4. $\log_{10} 10^x = x$
5. $\log_e x = \ln x$
6. $\ln e^x = x$
7. $\ln e = 1$
8. $\log_a x = \frac{\log_{10} x}{\log_{10} a}$

Differentiating Trigonometric Functions

1. $\frac{\partial}{\partial x}(\sin x) = \cos x$
2. $\frac{\partial}{\partial x}(\cos x) = -\sin x$
3. $\frac{\partial}{\partial x}(\tan x) = \sec^2 x$
4. $\frac{\partial}{\partial x}(\csc x) = -\csc x \cot x$
5. $\frac{\partial}{\partial x}(\sec x) = \sec x \tan x$
6. $\frac{\partial}{\partial x}(\cot x) = -\csc^2 x$

Chain Rule

1. Chain Rule: $[f(g(x))]' = f'(g(x)) * g'(x)$
2. $\frac{\partial}{\partial x}(u^n) = nu^{n-1} \frac{du}{dx}$
3. $\frac{\partial}{\partial x}[g(x)]^n = n[g(x)]^{n-1} * g'(x)$

$$4. \frac{\partial}{\partial x}(a^x) = a^x \ln a$$

Arc Rules

$$1. \frac{\partial}{\partial x}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$2. \frac{\partial}{\partial x}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Differentiating Log Functions

$$1. \frac{\partial}{\partial x}(\log_a x) = \frac{1}{x \ln a}$$

$$2. \frac{\partial}{\partial x}(\ln x) = \frac{1}{x}$$

$$3. \frac{\partial}{\partial x}(\ln u) = \frac{1}{u} \frac{du}{dx}$$

$$4. \frac{\partial}{\partial x}[\ln g(x)] = \frac{g'(x)}{g(x)}$$

$$5. \frac{\partial}{\partial x}(\ln |x|) = \frac{1}{x}$$

$$6. \frac{\partial}{\partial x}(a^b) = 0 \text{ when } a \text{ and } b \text{ are constants}$$

$$7. \frac{\partial}{\partial x}[f(x)]^b = b[f(x)]^{b-1} * f'(x)$$

$$8. \frac{\partial}{\partial x}(a^{g(x)}) = a^{g(x)} * \ln(a) * g'(x)$$

9.

Linear Approximations

1. Linear approximation of f at a: $L(x) = f(a) + f'(a)(x-a)$

Related Rates

1. Approach to solving related rate questions.
 - a. Always remember to read the question very carefully and make as many notes as possible
 - b. Draw a diagram to relate entities as often as possible
 - c. Introduce notation. Assign symbols to all quantities that are functions of time.
 - d. Express the given information and the required rate in terms of derivatives.
 - e. Write an equation that relates the various quantities of the problem. If possible, try to use the geometry of the situation to eliminate one of the variables by substitution.
 - f. Use the chain rule to differentiate both sides of the equation with respect to t.
 - g. Substitute the given information into the resulting equation and solve for the unknown rate.

Curve Sketching

1. Absolute Maximum: There is an absolute maximum at c if $f(c) \geq f(x)$ for all x in domain d.
2. Extreme values of a function f: These are the absolute maximum and minimum of a function f(x).
3. Extreme value Theorem: If f is continuous on a closed interval [a,b], then f attains an absolute maximum value f(c) and an absolute minimum value f(d) at some numbers c and d in [a,b].
4. Local maximum: There is a local maximum at c if $f(c) \geq f(x)$ for x near c.
5. Fermat's Theorem: If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.
6. Critical number: A critical number of a function f(x) is a number c in the domain of f such that $f'(c) = 0$ or $f'(c)$ does not exist.
7. If f has a local maximum or minimum at c, then c is a critical number of f.
8. Closed interval Method: To find the absolute maximum and minimum values of a continuous function f on a closed interval [a,b]:

Calculus 1 Cheat Sheet taken from MAT 1320 at the University of Ottawa.

- a. Find the values of f at the critical numbers of f in (a,b)
- b. Find the values of f at the endpoints of the interval.
- c. The largest of the values from steps 1 and 2 will give the absolute maximum and the smallest of the values will give the absolute minimum value.

Curve Sketching

1. The mean value theorem: if f is a differentiable function on the interval $[a,b]$ then there exists a number c between a and b such that $f'(c) = \frac{f(b) - f(a)}{b - a}$
2. The increasing / decreasing test:
3. The first derivative test:
4. The concavity test:
5. The second derivative test:

Quizzes

1. $\operatorname{cosec} \theta =$
2. $\sec \theta =$
3. $\cot \theta =$
4. $\tan \theta =$
5. $\sin^2 \theta + \cos^2 \theta =$
6. $\sec^2 \theta =$
7. $\operatorname{cosec}^2 \theta =$
8. $\sin(-\theta) =$
9. $\cos(-\theta) =$
10. $\sin(\theta + 2\pi) =$
11. $\cos(\theta + 2\pi) =$
12. $\sin(A \pm B) =$
13. $\cos(A \pm B) =$
14. $\tan(A \pm B) =$
15. $\sin 2x =$
16. $\cos 2x =$
17. What is the product rule?

18. What is the quotient rule?

19. $\frac{\partial}{\partial x}(\sin x) =$

20. $\frac{\partial}{\partial x}(\cos x) =$

21. $\frac{\partial}{\partial x}(\tan x) =$

22. $\frac{\partial}{\partial x}(\csc x) =$

23. $\frac{\partial}{\partial x}(\sec x) =$

24. $\frac{\partial}{\partial x}(\cot x) =$

25. What is the chain rule?

26. $\frac{\partial}{\partial x}(u^n) =$

27. $\frac{\partial}{\partial x}[g(x)]^n =$

28. $\frac{\partial}{\partial x}(a^x) =$

29. $\log AB =$

30. $\log \frac{A}{B} =$
31. $\log A^P =$
32. $\log 10^x =$
33. $\log_e x =$
34. $\ln_e x =$
35. $\ln e =$
36. $\log_a x =$
37. $\frac{\partial}{\partial x}(\sin^{-1} x) =$
38. $\frac{\partial}{\partial x}(\tan^{-1} x) =$
39. $\frac{\partial}{\partial x}(\log_a x) =$
40. $\frac{\partial}{\partial x}(\ln x) =$
41. $\frac{\partial}{\partial x}(\ln u) =$
42. $\frac{\partial}{\partial x}[\ln g(x)] =$
43. $\frac{\partial}{\partial x}(\ln |x|) =$
44. What are the three steps to doing logarithmic differentiation?
45. $\frac{\partial}{\partial x}(a^b) =$
46. $\frac{\partial}{\partial x}[f(x)]^b =$
47. $\frac{\partial}{\partial x}(a^{g(x)}) =$
48. Differentiate $y = x^{\sqrt{x}}$
49. Write the equation to a tangent line to a curve $y=f(x)$ at $(a,f(a))$.
50. Write the linear approximation or tangent line approximation of f at a .
51. I gave the following two terms in one of the study groups. Can you remember what they meant?
52. SIVIA:
53. ADVDS:
54. What are the steps that you use in order to solve related rate questions?
55. Define the following terms: absolute maximum of f , absolute minimum of f , extreme values of f . f is a function.
56. Define the following terms: local max, local min.
57. State the extreme value theorem.
58. What is Fermat's theorem?
59. What is a critical number in f ?
60. What is the Closed Interval Method and what is it used for?

61. What is the Mean Value Theorem? What does it remind you of?
62. Explain the increasing/decreasing test.
63. What is the first derivative test?
64. What do we use concavity for?
65. What is the concavity test?
66. What is the second derivative test?

67. What is L'Hospital's Rule?

68. What are the steps to follow when doing optimisation questions?

69. What is Newton's Method?

70. Give the definition of an anti-derivative.
71. Give the anti-derivative of $cf(x)$
72. Give the anti-derivative of $f(x) + g(x)$
73. Give the anti-derivative of x^n ($n \neq 1$)
74. Give the anti-derivative of $1/x$
75. Give the anti-derivative of e^x
76. Give the anti-derivative of $\cos(x)$
77. Give the anti-derivative of $\sin(x)$
78. Give the anti-derivative of $\sec^2(x)$
79. Give the anti-derivative of $\sec(x)\tan(x)$
80. Give the anti-derivative of $\frac{1}{\sqrt{1-x^2}}$
81. Give the anti-derivative of $\frac{1}{1+x^2}$

82. How do we approximate the area under a graph?
83. What is the difference between the LHR and the RHR?
84. The LHR and RHR give over-approximations and under-approximations under different conditions. Can you say what they are?

85. What is the midpoint rule?
86. $\int_a^b c dx =$
87. $\int_a^b [f(x) + g(x)] dx =$
88. $\int_a^b cf(x) dx =$
89. $\int_a^b [f(x) - g(x)] dx =$
90. $\int_a^b f(x) dx + \int_b^c f(x) dx =$
91. Three comparison properties of the general integral were mentioned in the text. What are they?

92. What is the evaluation theorem with respect to definite integrals?
93. What is the main difference when you solve definite and indefinite integrals?
94. $\int x^n dx =$

95. $\int \frac{1}{x} dx =$

96. $\int e^x dx =$

97. $\int a^x dx =$

98. $\int \sin x dx =$

99. $\int \cos x dx =$

100. $\int \sec^2 x dx =$

101. $\int \csc^2 x dx =$

102. $\int \sec x \tan x dx =$

103. $\int \csc x \cot x dx =$

104. $\int \frac{1}{x^2 + 1} dx =$

105. $\int \frac{1}{\sqrt{1 - x^2}} dx =$

106. What is the total change theorem?

107. What is the fundamental theorem of Calculus?

108. What is the substitution rule? (do it for both definite and indefinite integrals)

109. Integrals of symmetrical functions have special properties. Do you know what they are?

110. What is the formula for integration by parts?

111. Can you give a general explanation of the method of partial fractions?

112. $\int \frac{1}{x^2 + a^2} dx =$

113. What is the formula for the Midpoint rule?

114. What is the formula for the trapezoidal rule?

115. How do you calculate error bounds for the trapezoidal and midpoint rules?

116. What is Simpson's rule?

117. How do you calculate error bounds for Simpson's rule?