# MAT 1348 - Winter 2013
## - Solutions to HW#1 -

① Propositions:

$h$: "It is a holiday."
$s$: "There is lots of snow on the ground."
$m$: "You are able to go skiing on Mont Tremblant."
$w$: "You have lots to study."
$c$: "It is cold."

Compound propositions:

(a) $c \rightarrow s$
(b) $(m \wedge \neg w) \rightarrow h$
(c) $\neg m \leftrightarrow (\neg c \vee \neg s)$
(d) $(s \wedge \neg w) \rightarrow m$
(e) $(h \wedge s) \rightarrow (\neg w \rightarrow m)$

② (a) $A \equiv (p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$
$B \equiv (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ Alternatively: $B \equiv \neg p$

(b) $A \equiv (\neg\neg p \wedge \neg\neg q) \vee (\neg\neg p \wedge \neg q) \vee (\neg p \wedge \neg q)$
$\equiv \neg(\neg p \vee \neg q) \vee \neg(\neg p \vee q) \vee \neg(p \vee q)$

$B \equiv (\neg p \wedge \neg\neg q) \vee (\neg p \wedge \neg q)$
$\equiv \neg(p \vee \neg q) \vee \neg(p \vee q)$

(c) Given the equivalences in (b), we have that

$A \equiv \neg(\neg p \vee \neg q) \vee \neg(\neg p \vee q) \vee \neg(p \vee q)$
$\equiv \neg(p \rightarrow \neg q) \vee (\neg(p \rightarrow q) \vee \neg(\neg p \rightarrow q))$
$\equiv (p \rightarrow \neg q) \rightarrow ((p \rightarrow q) \rightarrow \neg(\neg p \rightarrow q))$

$B \equiv \neg(p \vee \neg q) \vee \neg(p \vee q)$
$\equiv \neg(q \rightarrow p) \vee \neg(\neg p \rightarrow q)$
$\equiv (q \rightarrow p) \rightarrow \neg(\neg p \rightarrow q)$

Note, solutions for any part of this problem are not unique.

(3) First we write the truth table:

| p | q | r | ¬q∨r | ¬p→(¬q∨r) | q→r | p∨(q→r) | (i) | ¬p↔q | (ii) | ¬p∨¬(p∨q) | (iii) | ¬(p∨q) | (¬p∨¬(p∨q))∧r | (iv) | lines |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | F | T | F | T | F | F | T | (1) |
| T | T | F | F | T | F | T | T | F | T | F | T | F | F | T | (2) |
| T | F | T | T | T | T | T | T | F | F | T | F | F | F | T | (3) |
| T | F | F | T | T | T | T | T | F | F | T | F | F | F | T | (4) |
| F | T | T | T | T | T | T | T | T | T | T | F | T | T | T | (5) |
| F | T | F | F | T | F | F | T | T | T | T | F | T | F | T | (6) |
| F | F | T | T | T | T | T | T | T | T | T | F | T | T | T | (7) |
| F | F | F | T | T | T | T | T | T | T | T | F | T | F | T | (8) |

(a) Proposition (i) is a tautology.
Propositions (ii) and (iii) are contingencies.

(b) Propositions (ii) and (iii) are false when

| p | q | r | (ii) | (iii) | line |
|---|---|---|---|---|---|
| T | F | T | F | F | (3) |
| F | T | T | F | F | (5) |

(c) Propositions (ii) and (iii) are logically equivalent.

(d) Yes, the set of propositions is consistent since for the assignment of truth values for p, q and r in lines 1, 2, 4, 6-8, all thre propositions (i), (ii) and (iii) are true.

(4) In the following, assume that

p: "A is a knight."
q: "B is a knight."

(a)

| p | q | A says: $\neg q \to F$ |
|---|---|---|
| (T) | T | (T) |
| T | F | F |
| F | T | T |
| (F) | F | (F) |

A says: $\neg q \to F$ (F = false)

We conclude that A and B are of the same type: both knights or both knaves.

(b)

| p | q | A says: $p \oplus \neg q$ | B says: $q \to \neg p$ | line |
|---|---|---|---|---|
| (T) | T | (T) | F | (1) |
| T | F | F | T | (2) |
| (F) | (T) | (F) | (T) | (3) |
| F | F | T | T | (4) |

A says: $p \oplus \neg q$
B says: $q \to \neg p$

Proposition p and what A says have the same truth value in lines (1) and (3).
Proposition q and what B says have the same truth value in the line (3).
∴ The solution is in the common line (3):
A is a knave and B is a knight.