STAT 2509B - Assignment #1

SOLUTION

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Q.1: [2]

a) (iii) population (1)

b) (iii) mean (1)

Q.2: [7] K - constant

X, Y - random variables

a) (i) $E(k) = k$ (1), (ii) $E(kX) = kE(X)$ (1),

(iii) $E(X \pm Y) = E(X) \pm E(Y)$ (1)

b) (i) $V(k) = 0$ (1), (ii) $V(kX) = k^2 V(X)$ (1),

(iii) $V(X \pm Y) = V(X) + V(Y) \pm 2\,Cov(X,Y)$ (1)

- if X & Y are indep. $\Rightarrow Cov(X,Y) = 0$

i.e. $V(X \pm Y) = V(X) + V(Y)$ (1)

Q.3: [8]

a) quantitative (1/2) & discrete (1/2)

b) categorical (1/2) & ranked (1/2)

c) quantitative (1/2) & continuous (1/2)

d) categorical (1) (or qualitative)

e) quantitative (1/2) & continuous (1/2)

f) categorical (1/2) & ranked (1/2)

g) categorical (1/2) & ranked (1/2)

h) quantitative (1/2) & continuous (1/2)

Q.4: [7] $N(\mu, \sigma^2)$; σ^2 known

a) (i) $\bar{x} \pm 1.96\, \sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 1.96 \Rightarrow \alpha/2 = 0.025$

$\alpha = 0.05$

$1 - \alpha = 0.95$

$\therefore$ 95% C.I. for μ (1)

(ii) $\bar{x} \pm 2.24\,\sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 2.24 \Rightarrow \alpha/2 = 0.0125$

$\alpha = 0.025$

$1-\alpha = 0.975$

$\therefore$ 97.5% C.I. for μ (1)

(iii) $\bar{x} \pm 3.09\,\sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 3.09 \Rightarrow \alpha/2 = 0.0010$

$\alpha = 0.0020$

$1-\alpha = 0.998$

$\therefore$ 99.8% C.I. for μ (1)

b) (i) $\bar{x} \pm z_{\alpha/2}\,\sigma/\sqrt{n} \Rightarrow 1-\alpha = 0.8968$

$\alpha = 0.1032$

$\alpha/2 = 0.0516 \Rightarrow z_{\alpha/2} = 1.63$ (1)

(ii) $\bar{x} \pm z_{\alpha/2}\,\sigma/\sqrt{n} \Rightarrow 1-\alpha = 0.9920$

$\alpha = 0.0080$

$\alpha/2 = 0.0040 \Rightarrow z_{\alpha/2} = 2.65$ (1)

(iii) $\bar{x} \pm z_{\alpha/2}\,\sigma/\sqrt{n} \Rightarrow 1-\alpha = 0.7540$

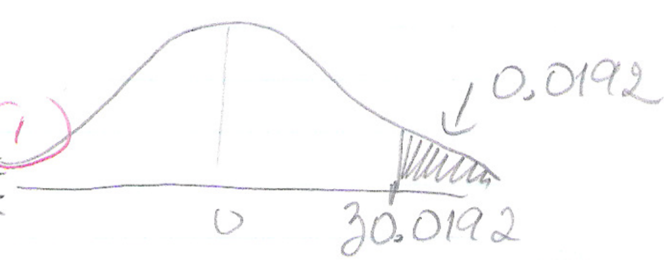
$\alpha = 0.2460$

$\alpha/2 = 0.1230 \Rightarrow z_{\alpha/2} = 1.16$ (1)

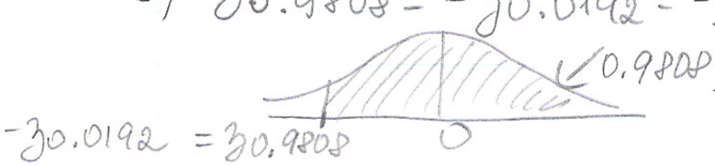
c) 90% C.I. would be narrower (1) than 99.20% C.I. since 90% would have shorter span (ie. it covers smaller interval of values).

Q.5 [6]

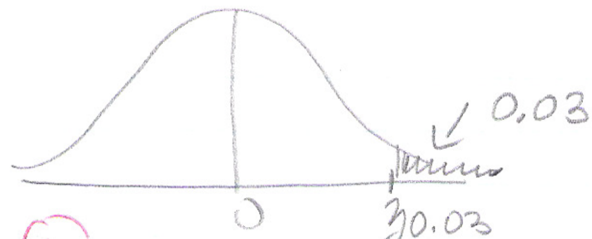
a) $z_{0.0192} = 2.07$ (1)



b) $z_{0.9808} = -z_{0.0192} = -2.07$ (1)



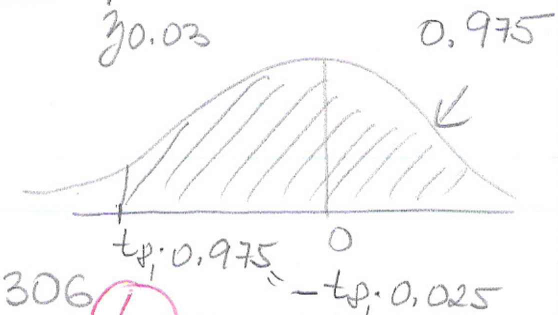
c) $z_{0.03} = \underline{\underline{1.88}}$ 1



d) $t_{8;\,0.025} = \underline{\underline{2.306}}$ 1

e) $-t_{8;\,0.025} = \underline{\underline{-2.306}}$ 1

f) $t_{8;\,0.975} = -t_{8;\,0.025} = \underline{\underline{-2.306}}$ 1



Q.6 : a) 2-sided hypothesis – is a 2-tailed test for testing parameter $\neq 0$ 1
[8]
(e.g. $H_0: \mu = 0$ vs. $H_a: \mu \neq 0$)

1-sided hypothesis – is a ~~1~~-tailed test for testing parameter < 0 1/2 or > 0 1/2
(e.g. $H_0: \mu \leq 0$ vs. $H_a: \mu > 0$
or $H_0: \mu \geq 0$ vs. $H_a: \mu < 0$)

- Steps involved:
1) state H_0 & H_a 1
2) test-statistic 1
3) rejection (critical) region 1
4) conclusion 1

b) • Type I error = error we make when we reject H_0 when it is true 1

P[Type I error] = α

• Type II error = error we make when we do not reject H_0 when it is false 1

P[Type II error] = β

Q.7: [6]

(i) μ - parameter (1)
(ii) σ^2 - parameter (1)
(iii) s^2 - statistic (1)
(iv) β_0 - parameter (1)
(v) $\bar{X}$ - statistic (1)
(vi) $\hat{\beta}_1$ - statistic (1)

Q.8: [4]

$$TSS = \sum_{i=1}^{n}(y_i - \bar{y})^2 \overset{?}{=} \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$$

Pf//

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 \overset{(1)}{=} \sum_{i=1}^{n}(y_i^2 + \bar{y}^2 - 2\bar{y}y_i) =$$

$$= \sum_{i=1}^{n} y_i^2 + \sum_{i=1}^{n} \bar{y}^2 - 2\bar{y}\sum_{i=1}^{n} y_i =$$

$$= \sum_{i=1}^{n} y_i^2 + n\bar{y}^2 - 2\bar{y}\sum_{i=1}^{n} y_i =$$

$$= \sum_{i=1}^{n} y_i^2 + \cancel{n}\frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n^2} - 2\frac{\sum_{i=1}^{n} y_i \left(\sum_{i=1}^{n} y_i\right)}{n} =$$

$$= \sum_{i=1}^{n} y_i^2 + \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} - \frac{2\left(\sum_{i=1}^{n} y_i\right)^2}{n} =$$

(2)

$$= \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$$

(1)